Thesis for the Degree of Ph. D.

"KANT'S VIEWS OF SPACE ABOUT 1769."

"Sich auf Ein Handwerk zu
beschraenken, ist das Beste.
Fuer den geringsten Kopf wird
es immer ein Handwerk, fuer
den bessern eine Kunst, und
der beste, wenn er Eins tut,
tut er alles, oder, um weniger
paradox zu sein, in dem Einen
was er recht tut, sieht er das
Gleichnis von allem was recht
getan wird."

Wilhelm Meister, Sechstes Kapitel.

Degree conferred 30th June, 1932.

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A. Commentators:

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2. Vaihinger: "Kommentar zur Kants Kritik der Reinen Vernunft." Stuttgart; Bd. I, 1881, Bd. II, 1892.

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6. Adickes: "Kant Studien." Kiel, 1895.

7. Kuno Fischer: "Commentary in Kant's Critique of Pure Reason." (Eng. Trans. Mahaffy.) Original, Heidelberg, 1869.

B. Kant's Predecessors who influenced him directly at this time in regard to the problems of space:

1. Clarke-Leibniz: "Controversy 1715."

2. Euler: "Essai sur l'espace et le temps." Berlin Academy 1748.

C. Kant's Writings relating to the views of space in this period.

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3. "Regions in Space." 1768. (Eng. Trans. Handyside) London, 1928.

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9. Kant Studien, 1925; Article by Siegel.

10. Gatterman: "Über das Verhältnis von Kant's Inaugural Dissertation vom Jahre 1770 zu der Kritik der reinen Vernunft." Halle, 1899.

11. E. von Hartmann "Kritische Grundlage." quoted by Vaihinger, II. p.228.

Synopsis:

(1). In his paper of 1768, Kant maintained that space is an absolute reality. (pp. 1 - 4.)

(2) In his study of the Clarke-Leibniz Controversy, which he undertook immediately after writing his paper of 1768, Kant discovered:

(a). Both Clarke and Leibniz maintained that the problem of the infinity of the world in space can not be given a necessary answer. (pp. 4 -5.)

(b) Clarke maintained that whether the world be finite or infinite, it exists in an infinite space, which is a real quantity, or a property of God. (pp. 11 - 13).

(c). Leibniz maintained that whether the world be finite or infinite, it is commensurate with space, which is an ideal order of the possible relationships of the bodies of the world to one another. (pp. 5 - 7.)

(d). Leibniz maintained that the world is necessarily infinite in quantity, but that it is possibly finite in respect to space. (pp. 9 & 10).

(e). Clarke showed that a world infinite in quantity and finite in respect to space is impossible, since Newtonian principles of motion will not hold in regard to it. (pp. 14-15).

(f). Clarke did not show that the world is necessarily infinite, and he left, therefore, the problem of extramundane space unanswered. (pp. 15-16.)

(3). Kant concluded from his study of the Clarke-Leibniz Controversy, that there is neither relative space nor absolute space, since

(a) Leibniz's view of the infinity of the world violated Newtonian principles. (pp. 16-18);

(b) Clarke's view of absolute space left the problem of extramundane space unanswered. (pp. 16-18).

(4). This is the equivalent of Diss. 15. D. without the argument based upon the nature of truth in Geometry (pp. 18-20).

(5). Kant turned to the nature of space as a concept of the understanding. (pp. 18-20).

(6). With the aid of Euler he discovered:

(a) The concept of space is not abstracted from sense (Diss. 15 A). (pp. 21-24).

(b) In contrast to the usual general concepts such as "extension" or "man", space is a singular representation (Diss. 15. B), a pure intuition (Diss. 15. C) and a form of intuition (Diss. 15. E). (pp. 24-37).

(7). In this fruitful line of thinking, he realized that the propositions in Geometry are synthetic and a priori, and used this fact to reenforce his arguments for space as a pure intuition and as a form of intuition. (pp. 35-37) and (pp. 43-47).

(8). Vaihinger is partly wrong when he says (II. 434), that in answer to the problem of the nature of judgments in pure Geometry, Kant made space a pure intuition; and in answer to the problem of the nature of judgments in applied Geometry, Kant made space a form of intuition. The doctrines of space were reached first independently of

the specific problems about the nature truth in of/Geometry. After the discovery that propositions in Geometry are a priori and synthetic, Kant used this fact in 1770 (as Vaihinger says) as his main argument for the doctrines of space as a pure intuition (in pure Geometry) and as a form of intuition (in applied). (pp. 37-42).

(9). Riehl and Fischer believe wrongly that Kant's main concern in 1768 was to show that space was an intuition. (pp. 42-43).

(10). Kant's introduction of the example of incongruent counterparts into Diss. 15. C. shows that he regarded that example as particularly significant in his doctrine of space as a pure intuition. (pp. 30-32).

(11). That doctrine may be developed just as well in contrast to any other particulars. Kant omitted the example of incongruent counterparts from the Aesthetic. (pp. 28-30).

(12). In the first paragraph to Diss. 15. cor., Kant contrasted the relation of the parts of space to the whole with the usual part-whole relation of determining part to determined whole. (pp. 48-49).

(13). In spite of his contrary view in the rest of of Diss. 15., Kant said at the end of Diss. 15. cor. that space is abstracted from an activity of the mind. The former view is more in line with teaching in the Aesthetic; the latter with the Analytic. (pp. 49-54).

(14). In Diss. 15, and in both the metaphysical and the transcendental expositions of space in the Aesthetic, Kant treated space as a conscious representation. (pp. 55-60).

(15). This point of view is in contrast to the original investigation of the concept of space in 1769, and in contrast to the later view of the Analytic. (pp. 55-60).

(16). Kant put two arguments in Diss. 15. A., in the one of which he treated sensation as a psychological process; in the other, as a psychological fact. They unite to show that space is non-empirical. (pp. 60-65).

a. He put the first argument in Aes. I. (pp 65-67).

b. He put the second in Aes. II. (pp. 67-68).

(17). In Diss. 15. A., and in Aes. I., and II., Kant was dealing with space as a conscious representation.

(18). The argument in Aes. II, which Kant took over from Diss. 15. A., is the doctrine of the relative necessity

of space for outer appearances. The doctrine of the absolute necessity of space is not in the Diss. Two main arguments in Aes. II. deal with these two types of necessity. (pp. 68-81.)

(18-a). The argument in Aes. I. and the doctrine of relative necessity in Aes. II., unite to show that space is non-empirical. (p. 74.)

(19). The doctrine of the absolute necessity of space means that space is a necessary representation, or that space is at the basis of all intuitive thinking. (pp. 68-74.)

(20). In spite of evidence to the contrary, the doctrine of relative necessity is not a corollary of the doctrine of absolute necessity. (pp. 74-81.)

(20-a). Similarly, pure Geometry does not include applied Geometry. (pp. 81-83.)

(21). In Aes. A. IV., B. III., Kant took over the argument of Diss. 15. B., omitting the problem of the boundlessness of space. He concluded from the contrast of the representation of space with the usual general concepts that space is non-conceptual, hence intuitive. (pp. 85-86.)

(22). He took over also the argument of Diss. 15. C.n. and Diss. 15. cor., (first paragraph), where he had contrasted the usual part-whole relation with the relation of the parts of space to the whole. (pp. 86-87.)

(23). He omitted the problem of the infinity of space until A. V. (pp. 87-88.)

(24). His contrast in A. IV., B. III., between "space" and "man" is a contrast between two conscious representations and not between logical abstractions. The whole of space determines the parts, which are limits. (pp. 88-90.)

(25). In the first sentence of A. V., Kant meant the contemplated infinity of space. (pp. 90-91.)

(26). In the argument of A. V., he showed only that space is conceived as infinite, and did not show that space is non-conceptual. His argument even leads to the opposite conclusion. (pp. 91-92.)

(27). In the first sentence of B. IV. he emphasized that he meant the contemplated infinity of space. In his new argument he showed validly that space is non-conceptual, but he did not show that space is a contemplated infinite whole. There are an infinite number of representations <u>in</u> space, not <u>under</u> it. (pp. 92-96.)

(28). In 1770 and in the Aes., he identified space, the pure form of intuition, with space, the pure intuition. He attempted to discover in the latter all of the characteristics of the former. (pp. 96-103.)

(29). In two parts of Diss. 15., Kant assumed that there is an activity of mind from which space is abstracted. (pp. 104-107.)

(30). There are three possible types of activity of the mind, one of which turns out to be meaningless:

a. An intellectual activity yielding a sensitive whole of space; (the doctrine of two parts of the Diss.);

b. An activity beginning with the sensitive and yielding a sensitive whole of space; (the doctrine of B. 160n). This doctrine is meaningless.

c. An activity beginning with the objects of sense (sensuous manifold) and yielding the sensitive whole of space; (the doctrine of the Analytic.) (pp. 107-108 and pp. 121-123.)

(31). Kant reconciled the opposing doctrines of space in the Diss. by means of a dualism between sense and intelligence, and stated the doctrine of the first type of activity. (pp. 108-111.)

(32). The second type of activity (assumed by Kant in B. 160n.) is meaningless. It is compatible neither with the Diss. and the Aes., nor with the Ana. (pp. 107-108); and (pp. 127-130).

(33). Only the third type of activity is compatible with the teaching of the Ana., and it is essential to that teaching. (p. 108); and (pp. 130-131). /The doctrine of this third type of activity contradicts the four arguments of the Aes. (pp. 137-139).

(34). Either Kant was aware of the conflict between the doctrines of the Ana. and the Aes., or it cannot be said that he grafted one upon the other. (pp. 111-112.)

(35). In the theory of knowledge which Kant developed after 1770 intuition and conception are united, both being necessary for knowledge. (pp. 112-113.)

(36). Knowledge in form (in pure Mathematics) is different, on the one hand from knowledge (in applied Mathematics), and on the other, from thinking, which is not knowledge. (pp. 113-114.)

(37). In pure Mathematics we do not have knowledge, but only knowledge in form. By means of knowledge in form we determine a priori the limits of knowledge. (pp. 114-117.)

(38). Space is determined by a synthetic activity of the understanding which must begin with the sensuous manifold. This activity yields a pure manifold (in pure Mathematics), and a pure form of the sensuous manifold (in applied Mathematics). (pp. 117-119.)

(39). Space is only the form for setting objects of sensation together. If the objects set together are removed, nothing remains. (p. 119.)

(40). Since the activity of the understanding, which is presupposed by space, begins with the sensuous manifold, sense-experience is prior to space. This contradicts Diss. 15. A. and Aes. I. (pp. 119-121.)

(41). Since nothing remains after abstracting from the objects of sense, space is not at the basis of external appearances. This contradicts Diss. 15. A., and A. II. (pp. 119-121.)

(42). Since the sensuous manifold is prior to space, the parts of space precede and determine the whole. This contradicts Aes. B. III. (pp. 119-121.)

(43). Since the sensuous manifold is prior to space, the particular representations of space are <u>under</u> it and not <u>in</u> it. This contradicts Aes. B. IV. (pp. 119-121.)

(44). The four arguments of the Aes. stand or fall with the dualism of 1770. (pp. 104-141, esp. p. 125.)

(45). In 1770, Kant did not face the question: how an activity of the mind could yield the concept of space. His thoroughgoing dualism makes this less unsatisfactory then than in the Critique. (pp. 124-127.)

(46). He formulated at B. 160n. the doctrine that space is preceded by a synthesis, which does not begin with the manifold of sense. (pp. 121-123.)

(47). Had he faced the question of how such a synthesis could begin, he would have realized that:

a. If there is a manifold other than the sensuous manifold, with which a synthesis can begin, then it must begin with the very thing which it is supposed to yield, namely, space. (pp. 127-130.) One particular space is "space" as much as any other particular space. (p. 129.)

b. The only kind of synthesis which begins with no manifold (pure or sensuous) is an intellectual synthesis, the possibility of which he had rejected with the dualism of 1770. (pp. 129-130.)

c. The only kind of synthesis compatible with his point of view in the Ana., is a synthesis which begins with the manifold of sense. (pp. 130-131.)

(48). His teaching in the Analytic (including B. 160n) contradicts his teaching in the four arguments on space in the Aesthetic. (pp. 130-131 and pp. 137-138.)

(49). Cassirer recognizes a unity in the diversity of the doctrines in the Critique. This unity is attained from the point of view in the Analytic. (pp. 131-132.)

(50). Kant's point of view in his first formulation of the doctrines of space in 1769 is nearer to the point of view of the Ana. than to that of the Aes. (pp. 132-134.)

(51). According to the point of view of the Ana., intuitions and conceptions cannot be separated, but they may be treated as separated, and knowledge can not run counter to the results of this treatment. (pp. 131-137.)

(52). In Diss. 15. cor. (at the end), Kant was correct in saying that there is an activity of the mind, but wrong as to the nature of that activity. (pp. 139-141.)

(53). In the rest of Diss. 15. (with the possible exception of the example of incongruent counterparts) and in the four arguments in the Aes., Kant was wrong to assume that there is no activity of the mind. (pp. 139-141.)

(54). Throughout Diss. 15. and the Aes., he was wrong to assume the dualism between sense and intelligence. (pp. 139-141.)

(55). In the light of the conflicting views of space in the Critique, it becomes clear that Kant could unify his doctrine of the contemplated whole of space and the activity of the mind in Diss. 15., only on the assumption of the dualism of 1770. (pp. 139-141.)

(56). With the rejection of that dualism after 1770, the arguments in Diss. 15., and in the Aes., require radical revision. Kant retained them in the Critique either because he was not aware of the contradictions involved, or because he saw how great an alteration is required to remove these contradictions. (pp. 139-141.)

P A R T I.

In 1768, Kant wrote his short paper, "Regions in Space."[1.] In it he accepted the doctrine of an absolute space. The example of incongruent counterparts [2.] shows that, in nature and in Geometry, certain relations exist which can be formulated only by reference to absolute primary space. A pair of human hands or two equal incongruent spherical triangles are alike in regard to internal arrangement; but one can not be superimposed upon the other. Their difference may be described only in reference to a space in which they are located. Kant agreed [3.] with Euler [4.] that the Leibnizian doctrine of space as a relation between bodies is inadequate to explain the laws of motion. Euler's critique of Leibnizian space is valid, but he did not prove that, to explain these laws, space must be an absolute reality. The evidence in Mechanics is evidence for the reality of absolute space only in a negative way. That is, it disproves that space is a relation, which Leibniz had maintained it to be; but it does not prove that it is an absolute reality. Kant's belief in Newtonian

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1.) English Translation, Handyside, 1929.
Original, Meiner, Phil. Bibliothek, 46b.

2.) See Handyside, (pp. 25-27.) for Kant's full statement of the problem of incongruent counterparts.

3.) Handyside, p. 21.

4.) Euler, "Essai sur l'Espace et le Temps." 1748; esp. Sec. V. VI.

Mechanics had led him to accept the absolute reality of space as far as required by the principles in that science. In 1768 he realized that he would have to adapt his position in every respect to suit their truth. His question was whether, in working out their implications, he would be forced to take the position that space was absolute, and prune away the difficulties from that doctrine, or whether some alternate account of the Newtonian position would do away with the necessity of absolute space. At the beginning of this period, he was well aware of these two possibilities. Either he must accept the concept of absolute space, and answer the criticisms of Leibniz [1.], or he must reject it, and show that Clarke [1.] had gone astray in accepting it. In either case, he was squarely accepting the principles of Newtonian Mechanics. In a search into the evidence for absolute space, he found the examples of incongruent counterparts in nature and Geometry. These show -- he held in 1768 -- that other types of knowledge give supporting evidence to the interpretation of Newtonian Mechanics which adopts space as an absolute reality. During the writing of his paper, he realized that the objections of Leibniz were unanswered,

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1.) See below p. 8 and p. 9.

and that they would have to be considered at once. He knew that his thought had only been thrown tentatively in the one direction, rather than in the other by this new evidence, but he was resolved first to formulate his new conclusions. The hypothesis of his paper was the reality of absolute space. Could this be retained? To see whether he was on the right track would require immediate testing. The basis for this process would be the tenets of Newtonian Mechanics, and the arguments built up for and against absolute space by previous protagonists, mainly Leibniz and Clarke. He closed his paper, therefore, with an admission of the difficulties of the doctrine which he was adopting in spite of the evidence which his examples carried with them, and turned to the Controversy of Clarke and Leibniz, thinking of the main argument of Leibniz[1.] against absolute space, namely, that such space involves space outside of the universe, if the universe be finite. This study led him soon to realize that the problem of extramundane space involves the questions concerning the infinity of the world. If the world be infinite, does that mean infinite in space? If the world be infinite in

1.) Leibniz: 5, 29. The numbers cited in reference to Leibniz and Clarke refer to Letter and Section, respectively, of their correspondence.

space, does not that solve the difficulty of extramundane space, because there can be none? If the world be finite in space, is there extramundane space, or does space end finitely with the world? What light would a renewed study of the Controversy of 1715 throw on these questions?[1.) 3.)]

Is the world finite or infinite in respect to space and time? In his study of the Clarke-Leibniz Controversy, Kant found that neither adversary took the position that the world is necessarily finite or infinite. What is there in the nature of this question, which forced both Leibniz and Clarke to refrain from a necessary answer? Why could each of them speak of the possibility of the world being either finite or infinite, and then go on to draw conclusions about space and time and the world from both possible points of view? If there can be no necessary answer, from which point of view was this fact correctly established? Clarke's position was that whether the world be finite or infinite, space and time are necessarily real and infinite.[2.)] Leibniz's position was that whether the world be finite or infinite, it coincides necessarily with space and time.[4.)] These

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1.) Vaihinger II. pp. 436, 529-531; Cassirer, II p. 621n.
2.) Clarke 4, 7; 5, 41.
3.) For the full titles of the commentaries referred to, see Bibliography, above pp. b-d.
4.) Leibniz 4, 41: 5, 47.

positions are opposing but not contradictory. It is possible to hold with Clarke that space and time are necessarily infinite, and, at the same time, to hold with Leibniz that the world coincides necessarily with it. This is a position which both claimed to have avoided. Yet Clarke said Leibniz had taken it, and Leibniz said Clarke had done so.

Leibniz's position in regard to these questions was as follows: He made space and time solely a relation of things (bodies) to one another. For him the only reasonable questions concerning the beginning of the world are questions concerning how bodies actually began.[1] He thought that Clarke's main error was his acceptance of an absolute real space. By this acceptance such a meaningless question is raised as why the world had not begun a number of years sooner. Clarke's position led inevitably to one of two conclusions, Leibniz maintained. Either no world exists, or it has been created before any assignable time, and is necessarily eternal. By parity of argument, it is also necessarily infinite. Only by regarding time and space as relations between things, and non-existent without them, is this confusion avoided.

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1. Leibniz 4, 15.

Is the world eternal? This was for Leibniz a meaningless question when referring to anything else than a relation of successive things.[1.] Is the world infinite? This was meaningless when referring to anything else than a relation of simultaneous things.[1.] If space be absolute and real apart from things related, then any existing world is necessarily infinite. Leibniz believed that on Clarke's premisses the world has to be accepted as necessarily infinite, or its existence has to be denied. From his own premisses, Leibniz believed that such a choice is not necessary. Space and time are, for him, coincident with the world. They are ideal orders containing the possibility for the relationship of bodies among themselves as coexistent or successively existent. As such, they lose all meaning apart from bodies and are imaginary. The question of the infinity of the world according to space and time concerns only the infinity of these relationships. It is not a necessary question. It is not necessarily one question, or several parallel questions.

1.) Leibniz 4, 41. Space "est cet ordre qui fait que les corps sont Situables, et par lequel ils ont une Situation entre eux en existant ensemble, comme le temps est cet ordre par rapport à leur position successive."

To ask whether coexisting things are related infinitely, is not to ask whether successive things are related eternally. To ask whether successive things are related eternally a parte ante is not to ask whether they are similarly related a parte post.[1] None of these are necessary questions, and the answer to one of them in no way affects the answers to the others. They vary in reasonableness.

In his study of the controversy in 1769, Kant tried to combine Leibniz's view of the infinity of the world[2] with his own position of 1768 that there is absolute real space. His thinking divided itself into two parts. On the one hand, he was defending the concept of absolute space against Leibniz's question in regard to extramundane space. He regarded all arguments for absolute space unsatisfactory unless they answered that question. On the other hand, he was testing the concept of relative space with the requirements of Newtonian

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1.) Leibniz 5, 74. "Quand l'étendue de la matière n'auroit point de bornes, il ne suit point que sa durée n'en ait pas non plus ---. De plus, le commencement du monde ne deroge point à l'infinité de sa durée a parte post, ---"

2.) R. 1417 --- "Wir müssen uns aber beim Anfange aller Dinge wol eine Relation derselben a parte post gedenken, aber keine a parte anti, also kein Verhaltnis der Welt zu einer vorher verflossenen Zeit." Also R. 1416, 1419. Adickes p. 124-125; Cassirer II, p. 62ln. (R. means Reflexion, Erdmann's numbering.)

principles. He asked whether Leibniz's doctrine of the infinity of the world was satisfactory. If this view were found to be compatible with Newtonian teaching, the attack of Clarke upon the concept of relative space would have been found to be unsuccessful; and, if, in the meantime, Clarke had failed to answer Leibniz's problem with reference to extramundane space, Kant would be forced to reject the concept of absolute space, and to accept that of relative space in its stead. If Leibniz's doctrine of the infinity of the world were found, however, to be incompatible with the laws in Mechanics, the attack of Clarke upon the concept of relative space would have been found to be successful; and, if, in the meantime, Clarke had answered Leibniz's problem with reference to extramundane space, Kant would be forced to reject the concept of relative space, and accept that of absolute space in its stead. Kant searched into the arguments of Clarke and Leibniz with these two possibilities in mind. The light of 1769 [1.] had not dawned. He was testing Clarke's defense of absolute space in the face of Leibniz's attack. Would that defense answer the question

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1.) R. 4.

about space beyond the world? He was testing Leibniz's defense of relative space against Clarke's attack. Would Leibniz's view of the infinity of the world withstand the evidence of the truth in Mechanics? The third possibility, which he did not yet see, was that neither defense would succeed. Neither the concept of absolute space, nor the concept of relative space would bear the destructive criticisms wielded against it. Kant was soon to find that Clarke destroyed completely Leibniz's view of the infinity of the world, but failed (like Euler) to defend the concept of absolute space against Leibniz.

The term "infinite" was used in two senses by Leibniz in the Controversy. In regard to quantity or mass, he believed the universe to be indeterminate. That is, no quantity of matter may be assigned to it. In this sense, the universe is necessarily infinite, or indefinite. He had in mind a sort of maximum and minimum density, the absolute "plenum", and the absolute "vacuum", neither of which, from his point of view, exists. It is impossible to set a fixed quantity of matter between them, and say that

it is the quantity of the universe.[1] Somewhere between exists the matter of the universe, but no bounds may be set to it. It may be given no assignable quantity. There is no such thing as the amount of matter in the universe. In sharp contrast to this view of the "infinite", he considered infinite duration and extension. The indeterminate quantity of matter in the universe involves in no way, infinity of duration or extension. In his view, there is necessarily no such thing as the amount of matter in the universe. But this means in no way that the universe is necessarily either infinitely extended or enduring. It is possible that, in the latter respects, the world is either finite or infinite. Clarke, on the other hand, believed that these two sorts of infinity are inseparable. From him, there is only one sort of infinity, and this includes mass, and extension and duration. A world of infinite mass with finite extension and duration is impossible since Newtonian principles[2] of motion will not hold in regard to it. Either the world is infinite in every respect or in none. In the latter case, its mass, extension and duration

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1.) Leibniz 4, 21. "Il n'y a point de raison possible qui puisse limiter la quantité de la matière. Ainsi cette limitation ne sauroit avoir lieu." 4. Postscript. "Il n'est point possible qu'il y ait un Principe de determiner la Proportion de la matière, ou du rempli au vuide, ou du vuide au plein."

2.) Clarke 5, 73-75; see below p. 15n.

would be calculable, as well as its motion in space and time. It was Leibniz's double treatment of the problem of the infinity of the world which brought his position into conflict with Newtonian principles. To speak of a finite world whose mass is incalculable, is contrary to the meaning of "finite" in Mechanics. In that science, a finite body is one whose mass is calculable. If there be finite bodies whose masses are not calculable, then their motions likewise cannot be calculated. Clarke and Kant tested Leibniz's position with Newtonian principles, and found in it this serious flaw.

Clarke's position in regard to the questions of the infinity of the world was as follows: He said that it is possible for the world to have begun sooner or later than it actually began, and that it is possible for it to end sooner or later than it will end, because it is in infinite space and eternal time.[1.] Infinity and eternity contain an eternal number of possible beginnings and endings, and possible sizes. From

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1.) Clarke 4, 15. "It was no impossibility for God to make the World sooner or later than he did; nor is it at all impossible for him to destroy it sooner or later than it shall be destroyed ....."

his point of view, if matter and space (bodies and space) be the same, then the world is necessarily infinite and eternal, because space and time are infinite and eternal. If the world be coincident and coextensive with time and space, it shares their eternity and infinity. He referred here to Leibniz's position that space is the relation between things as commensurate with them, and dependent upon them for existence. From his own point of view --- Clarke maintained --- the necessity of considering the world infinite and eternal is avoided, because the other possibility remains of having the world begin and end quite finitely within eternal time, and also have finite bounds within infinite space. But admittedly, this possibility would entail an acceptance of extramundane space and time.[1.] Clarke denied Leibniz's charge that he must accept the world as necessarily infinite, and charged that Leibniz must accept it as necessarily infinite.

Clarke in no way gave up the possibility of the world's being infinite and eternal, that is, of the world's

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1.) Clarke 4, 7. "Extramundane Space (if the material world be finite in its Dimensions), is not imaginary but real." (Also, 5, 46.)

being infinite and eternal in infinite space and eternal time. He neither necessarily affirmed nor denied that position, for he was concerned only to show that space and time are infinite real quantities or properties. In his language, the world is dependent upon the will of God; that is, it is possible for it to be eternal, or to have begun and to end at any moment in an eternity of time. It is possible for it to be infinite, or to be bounded at any point in an infinity (immensity) of space. But time and space themselves are dependent upon the essence (existence) of God; they are properties of his essence, or effects of his essence.[1.] Far from it being possible for time to have begun or to end at any moment in eternity, or for space to be bounded at any point in infinity (immensity), time and space are eternity and immensity in which such bounds are set. Whether there are bounds set to the world, or whether it is eternal and infinite, and coincides with time and space, concerns only the will of God, and no necessary answer can be given; but time and space themselves are necessarily infinite because they concern His very existence. Any judgment about the will of God is a contingent judgment.

1.) Clarke 4, 15; 5, 45.

Any judgment about His essence is a necessary judgment. If there be not coincidence, Clarke believed, there has to be space and time empty of matter, although possibly containing other substances which are not matter.[1.)]

Kant saw that only Clarke's treatment of the problem of the infinity of the world is compatible with the laws in Mechanics. The contradiction in Leibniz's point of view is in regard to the nature of infinity. If the universe be quantitatively boundless, it cannot possibly be finite in respect to space and time. Clarke showed this by reference to Newtonian principles of motion. If the world be finite in respect to space and time, it is moveable according to those principles. But if the world be boundless in quantity, it is infinite in inertia, and it is immoveable. Leibniz was forced, therefore,-- Kant realized--- either to admit that the world is necessarily finite in quantity (mass) and space and time, in which case its motion is calculable; or hold that the world is necessarily

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1.) Clarke 4, 9; 4, 10.

infinite in respect to space and time as well as in quantity. Leibniz claimed that his view allowed both the infinitude and the finitude of the world as possibilities, despite his claim that it is necessarily boundless in quantity. But Kant realized that Leibniz was forced either to accept its necessary infinitude in every respect or admit that a finite quantity of matter may be assigned to it.[1.] This was exactly Clarke's position. It was, therefore, Clarke's view which alone allowed him satisfactorily to accept both the finitude and the infinitude of the world as possibilities. Since only Clarke's view was properly grounded, it was from that view that a choice between the two possibilities had to be sought. He found that only Clarke's account of the infinity questions was compatible with the facts in Mechanics. He clearly formulated Clarke's position. Either the world is finite and moveable in an infinite real

1.) Clarke 5, 73-75. "If the Material Universe can possibly --- be finite and moveable --- then Space (in which that Motion is performed) is manifestly independent of matter, but if on the contrary, the material Universe cannot be finite and moveable, and Space cannot be independent upon matter; then -- it follows evidently that God neither can nor ever could set bounds to matter, and consequently that the material Universe must be not only boundless but eternal also, both a parte ante and a parte post."

quantum of space, or it is infinite and immoveable in the same. Which? If it be finite, what is the nature of the real space beyond it which contains no bodies? To prove its necessary infinity would at least make unnecessary an answer to this question. Yet Clarke offered no way of proving either its finitude or its infinitude. He offered only the consolation, that, from his point of view, either is compatible with the truths in Mechanics.

Kant's study of the Correspondence between Clarke and Leibniz was complete. Yet it had not been unfruitful. Both Clarke and Leibniz claimed that the question concerning the infinity of the world can not be given a necessary answer. Yet Clarke alone is justified in making this claim. Leibniz's view is incompatible with the principles in Mechanics; his view of relative space is untenable. But Clarke did not prove that the world is necessarily infinite; he did not claim to do so.[1] Leibniz's criticism with regard to the problem of extramundane space is therefore quite valid.[2] Kant's study of the Correspondence

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1.) See above 15. n.; Clarke 5, 73-75.
2.) R. 1418.

was complete because he recognized finally the nature of the difficulty in that dispute. The light of 1769 had dawned.[1.] He found that both Clarke and Leibniz had succeeded, and failed. Each had succeeded in destroying the view of the other. Each had failed to maintain his own view. Clarke had shown that the view of space as a relation between existing objects is untenable, because the view of the infinity of the world to which it led is incompatible with Newtonian principles. Leibniz had shown that the view of space as an absolute reality is untenable, because, not proving that the world is necessarily infinite, it involved the possibility of space beyond the world. There is neither absolute space, nor relative space. In 1769 Kant reached the same conclusion about Clarke and Leibniz that in 1768 he had reached[2.] about Euler. Now (in 1769) he was forced to include his own view of 1768 in his criticism. Clarke and Euler and Kant, himself, in his own paper of 1768, had professed to make a positive argument, and had succeeded only in making an effective negative one. They had proved con-

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1.) R. 4.
2.) Handyside p. 21.

clusively that space is not a relation, but they had not proved that it is an absolute reality. Likewise, Leibniz had professed to make a positive argument, and had succeeded only in making an effective negative one. He had proved conclusively that space is not an absolute reality, but he had not proved that it is a relation. The arguments of Kant and Clarke and Euler had broken down before Leibniz's argument in reference to extra-mundane space. The arguments of Leibniz had broken down before Clarke's view of the problem of the infinity of the world. They had all four failed because space is neither an absolute reality, nor a relation between existing bodies.

What is it? Kant turned his attention simply to the question: What is the nature of space, as a concept of the understanding?[1.] With the aid of Euler's criticism of the doctrine[2.] of relative space, Kant sought the answer to this question. Kant left the problem of the infinity of the world as containing no necessary answer. Space is neither an absolute reality, nor a relation between objects. He considered the nature of space, as a concept of the understanding.

1.) Adickes, p. 126; Cassirer, II. p. 622n.
2.) Op. Cit., Sec.XIV., XV.

To understand how Kant reached his doctrines of space as a pure intuition and as a form of intuition in 1769, the distinction must be kept in mind between his earliest formulation of these doctrines and his formulation of them with the reenforcing evidence gained from the nature of judgments in Geometry. Both of these formulations were made in 1769-70. The first was a direct outcome of his study of Euler, and is found in those portions of the Dissertation which base the doctrine of space as a pure intuition directly upon the nature of the concept of space. The second is the more significant doctrine, and is the result of the insight (gained by Kant during or just after the first formulation) that the judgments in Geometry are a priori and synthetic.[1.) 3.)] This later formulation is found reenforcing every paragraph of Sec. 15 of the Dissertation[2.)] except paragraphs A. and B.

1.) Vaihinger, I. 269-275, deals with the development by Kant of the difference between "synthetic" and "analytic" judgments. In 1770 for the first time Kant recognized that the judgments in Geometry are synthetic and a priori. Vaihinger I. p. 274. "In den Grundsaetzen der Geometrie, in der Construction der Postulate, in allen Beweisen ist es Anschauung, welche die Mathem. Saetze vermittelt, nicht begriffliche Analyse. Nun war es aber auf Anschauung beruht, so ist sie --- empirisch. Somit wird die Endeckung ergaenzt durch den Nachweis, dass die Anschauung, welche der Mathem, zu Grunde liegt, eine reine, d. h. apriorische sei."

2.) Inaugural Dissertation, 1770. Meiner, Bd. 46 b; Handyside, pp. 59-66.

3.) See above, p. 4, footnote 3.

The results of Kant's thinking in 1769, up to his earliest consideration of the concept of space as a concept of the understanding, coincide roughly with Diss. 15. D.,[1.] with the important difference that in 15. D. the evidence which Kant gave to show that space is neither a substance nor a relation is supported by the nature of truth in Geometry. At the close of his study of Clarke and Leibniz, Kant saw that space is neither an absolute reality, nor a relation between real objects. He got his conclusion from that study. It is compatible, on the one hand, with Clarke's view that the world is neither necessarily finite nor infinite, and, on the other, with Leibniz's view that there can be no space beyond the world, that is, beyond particular bodies. What, then, is the nature of the concept of space? Kant's next step was taken from Euler's[2.] attack upon Leibnizian space.

1.) An adequate understanding of my discussion from this point on requires a constant reference to the Inaugural Dissertation, Sec. 15.

2.) Op. Cit.

Euler, in his paper,[1] was defending the concept of an absolute real space. In Sections XIV.[2] and XV. there was, however, a significant difference from the main body of his proof. Whereas, in the main body, he fell back upon the principles in Mechanics as evidence for the nature of space; here he took the offensive against Leibniz in regard to the manner in which the concept of space is reached or obtained. In Sec. XIV, Euler showed Kant three facts. First, general concepts are not derived from the senses.

> "Il est vrai que les sens ne sont pas capables de nous fournir les idées abstraites, semblables aux idées des genres & des espèces, qui n'existent que dans notre entendement, & auxquelles il ne repond aucun objet réel."

Second, the way in which the concept of space is reached is different from the way that general concepts are reached.

> "--- la manière, dont on parvient à l'idée de l'espace & du lieu, est bien differente de celle, dont nous nous formons les idées des genres & des espèces."

Third, it is wrong to claim that only those things exist,

1.) Essai sur l'Espace et le Temps.
2.) My references to Euler and to Kant's Dissertation must not be confused with each other. Only the references to Euler are to sections in Roman numerals.

the concepts of which are obtained through reflection.

> "Et on se tromperoit fort, si l'on vouloit soutenir, qu'il n'existe pas des choses, dont nous n'avons d'autres idées que par réflexion.

It is obvious from the analysis of this argument that Euler had in mind two ways in which a concept can be obtained, (a) from the senses; (b) through reflection. He was certain that the concept of space is not obtained in the way a general concept such as that of "extension" or of "man" is obtained. He was certain that a general concept such as that of "existence" or of "man" is not obtained from the senses. He did <u>not</u> maintain that the concept of space is obtained from the senses. How is it obtained, if neither through reflection, nor from the senses? In Sec. XV., he continued his argument.

The concept of extension is formed by taking away from the idea (concept) of a body all of the determinations except that of the extended.

> "C'est ainsi que nous nous formons l'idée de l'étendue en général, en retranchant des idées des corps toutes les determinations, hormis l' étendue."

But --- he went on to say--- the concept of place (whose

nature he treats parallel to that of space) which a body occupies, is obtained quite differently. It is gained not by taking away determinations from a body, but by taking away the body itself with all of its determinations. Place (and space) remain after this has been done, and therefore they can not be determinations.

> "Mais l'idée du lieu qu'un corps occupe, ne se forme pas en retranchant quelques determinations du corps; elle resulte en ôtant le corps tout entier; de sorte que le lieu n'ait pas été une determination du corps, puisqu'il reste encore, après avoir enleve le corps tout entier avec toutes ses qualités."

He concluded with the statement that the place (space) which a body occupies is quite different from the extension, which belongs to the body, and passes with it from place to place; whereas, place and space are immoveable.

> "---l'étendue apartient au corps, & pass avec lui par le mouvement d'un lieu à l'autre; au lieu que le lieu & l'espace ne sont susceptibles d'aucun mouvement.

From this section of Euler's paper Kant formulated the following questions, not about absolute space, but about space as a concept, a concept of the understanding. (1). Is the concept of space abstracted from sense?

(2). Does it include particular spaces under it as a general concept includes particulars? (3). How is the concept of "space in general", related to the concept of a particular space? (4). How is the concept of space related to an object in it? (5). How do these relations differ from the usual relation of a general concept such as "man" or "extension" to a particular such as "Peter" or "the extended"? Kant stated first his purely negative argument that the concept of space is not abstracted from sense.[1] It is presupposed by the objects of sense which it contains. He then turned to the further questions, the answers to which showed him that the concept of space is a singular representation;[2] and more definitely, a pure intuition[3] and form of intuition.[4]

The discovery that the concept of space is a singular representation, and a pure intuition, was made by contrasting it with general concepts. What is the nature of this contrast? First, there is "space in general." What is the nature of "space in general"? Is it like other general

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1.) Diss. 15 A. For an analysis of this section and discussion of its relation to the Aesthetic see below pp. 60-68.
2.) Diss. 15. B. Kemp Smith, p. 81. "Representation (Vorstellung) Kant employs in the widest possible meaning. It covers any and every cognitive state."
3.) Diss. 15. C.
4.) Diss. 15. C. and E.

concepts? Or is it different? Second, there is "this particular given space". Is "this particular given space" like other logical particulars? Third, there is "a particular thing in a particular space". What is "a particular thing in space" in terms of Kant's new doctrine? What is the logical significance of the phrase "in space"? What sort of relation does it indicate? It was the attempt to relate these concepts logically that led to Kant's development of his doctrine of the ideality of space beyond the point where space is a concept of the understanding. Euler showed him how the relations between space and a particular space, between a particular space and the object in it, differ from the relation of any general concept, such as "man" or "extension", and a particular, such as a particular man or extended object.

Take the general concept "man". It contains less than any individual man contains. Each individual man is more than the common denominator called "man". Peter is not meant every time a man is referred to. On the other hand, "space" is not qualitatively less than "an individual space". "An individual space" contains no more space-quality than the general concept "space" does. There are no characteristics of particular spaces which have to be abstracted from them in order to obtain the general concept of "space". The only way that "this space"

differs from "that" is that "this" is here, and "that" there. That this space is here, means no more than that this space is this space. "Here" and "there" are not qualities of space.[1] They are space itself. Euler showed Kant that "space in general" is not related to every particular space as a general concept to a particular. It is not the relation of the general concept "extension" to the particular extended thing. Kant called the relation a singular relation of "space" to "this space." Space is a singular concept or representation.[2] In regard to its characteristics, "space in general" contains no less than "this space" contains. Peter and Paul are both men; but they do not contain the same amount of "man". "This space" and "that space", however, have exactly the same amount of the character of space. "This given space" has neither more nor less space-quality (that is, characteristic or quality of space) than "space in general".

1.) See below pp. 91-92.
2.) Diss. 15 B.

"Space in general" seems no more to be given to the mind than "man in general". Yet it is immediately known (that is, without the mediacy of abstracting qualities). Because of the immediacy of this knowledge, "space in general" is given to the mind, whereas "man in general" is not. Knowledge of general concepts directly through their particulars is impossible. In the case of "man" and of "space" there are existing particulars. Peter, and Paul, or "this space" and "that space" are existing particulars. Exact conclusions about space are immediately or directly attainable. Knowledge about Peter is not ipso facto knowledge about"man". But knowledge about a particular space is ipso facto knowledge about space. The immediacy of this knowledge means that "space in general" is given to the mind. Both particulars, Peter and "this space", are so given. "Space in general" is given to the mind, but "man in general" is abstracted or derived. Truth about "space in general" is necessary truth, because it lies directly in the given concept. Truth about "man" is derivative truth, dependent upon qualities abstracted from particular men. "All men are mortal"; this truth is contingent upon the

number of individual men under observation. After millions of mortal men have been observed, an immortal man is not logically impossible. On the other hand, truth about space is a necessary truth. An immediate truth about "this space" can not be contradicted by truth about "any other particular space." Truth about the whole of space precedes truth about the given parts. Truth about space is not truth derived from qualities of "this space". It is given with "this space" or with space. The kind of concept that yields such a priori necessary truth is a singular representation or pure intuition.

The example of a right hand and a corresponding left hand is no more significant than the example of "Peter" and "Paul", except in one way. In both examples, when the two particulars are compared with two particular spaces, space is found to be a pure intuition. In either example, the comparison shows that "space in general" is given with "this particular space", whereas "hand" or "man" is only derived. In either example, it is clear that knowledge of space is immediate in contrast to knowledge of "hand" or "man". The example of a pair of hands is, however, different in one way. Whereas the contrast of "Peter" and "Paul", or two extended

objects, or any other pair of particulars, with two particular spaces, shows that space is an intuition, the example of "a right hand" and its partner "a left hand" not only shows this, but also forces the mind to make or construct space, in its attempt to make clear the difference between the hands. In attempting to take away the object (hand) and all of its determinations, the determination of right (or left) handedness is unaccounted for without the intuition of space. Unless space remains (as Euler said it did)[1.], the account of the nature of the two hands is incomplete. For this reason, Kant used in Diss. 15 C. the example of incongruent counterparts to show specifically that space is a pure intuition, and formulated the more general argument in Diss. 15 B., where he showed that space is a singular representation, and in Diss. 15 D., which is mainly the critique of Clarke and Leibniz. As I have shown above, the consideration of space in contrast to any other general concepts is evidence that space is a pure intuition, while the example of incongruent counterparts has an advantage only because in it the mind is forced to construct space. This advantage, together

1.) Euler Sec. XV.; see above p. 22-23.

with the knowledge that the axioms and propositions in Geometry are a priori and synthetic, which Kant gained also in 1769, [1.] caused him to use only the examples in Geometry and the example of incongruous counterparts (which is, of course, also an example in Geometry) in the main proof (Diss. 15.C.) that space is a pure intuition. His own first formulation of that doctrine, after which he gained a more adequate insight into the nature of the axioms and propositions in Geometry, was the much wider procedure of setting the concept of space into contrast with any general concepts.

Kant's selection of the example of incongruent counterparts left him, however, open to one danger. If, in proving that space is a pure intuition, he required an example which forces the mind to construct space psychologically, he raised the question whether the doctrine of space as a pure intuition is a doctrine based upon the nature of space in contrast to general concepts, or whether that doctrine is an account of the activity of the mind (excited by sensation) in the construction of space. The doctrine, as Kant first developed it, is a denial of the activity of the mind either as abstracting from

1.) See below p. 36.

sensation[1] or as a reflective process.[2] But Euler[3] had stated that the concept is reached in a way different from that in which general concepts are reached. And in the midst of the formulation of the doctrine that space is a pure intuition, Kant was already enough occupied with the question of how space is reached or constructed to base his proof upon the example of incongruent counterparts, which forces the mind to <u>reach</u> or <u>construct</u> space, in a special way, rather than upon the contrast between a pure intuition which is contemplated[4] and general concepts which are abstracted. Kant makes this departure from the more exact formulation of his doctrine even more significantly at the end of the Corollary to Diss. 15., where, in spite of his previous doctrines, he raised the question, whether the concept of space is innate or acquired. There he held actually that it is acquired, and stated that an act of mind constructs it. Obviously, in regard to <u>this</u> view, the example of incongruent counterparts is of unique significance, since it forces the mind to make a construction. From Kant's stricter point of view, space is not constructed but contemplated. It was to this view that the thinking of 1769 led,

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1.) Diss. 15. A.
2.) Diss. 15. B.
3.) See above p. 21.
4.) In Diss. 15. C., he spoke of the <u>contemplation</u> of the pure intuition. More exactly, it would be the contemplation of space.

and it was this view that was supported in 1769-70 by the fact of the a priori synthetic nature of propositions in Geometry. As we shall see, this view was in line with his development in the Aesthetic,[1.] while the view that the mind constructs space, (potentially the view in the example of incongruent counterparts, and expressly the view in the Corollary to Diss. 15.) became even more directly stated in the Analytic.[1.] Our present problem, however, is to trace Kant's development of the view of space as a form of intuition, and his awakening to the nature of judgments in Mathematics.

The first of Kant's logical problems had been solved. He found that the relation between space and "this particular space" is a singular relation. Space is a pure intuition. Truth about space is given immediately with "this space". It is not knowledge gained by abstracting qualities from "this space", as knowledge of "man" is gained by abstracting qualities from Peter. The unique nature of this truth is its a priority. Truth about "man" gained by abstracting qualities from "Peter" is a posteriori. The distinction concerns a relation between concepts. Kant's problem of the relation of space to this particular space had been solved.

1.) A section of the Critique of Pure Reason.

There remained for him the problem of the relation of "this particular space" to "this particular object in it".

Kant's explanation of the relation of "this particular space" to "this particular object in it" came also directly from Euler. By means of the doctrine of space as a pure intuition, Kant had explained the relation of "space in general" to "this particular space". In solving the second of his logical problems, he could speak of both "space in general" and "this particular space" as "space", since they are the same. His second problem was the relation of space to the particular object in it. What is the nature of the relation expressed by the preposition "in"? It is not the relation of "extension" to "the extended". An extended object is not in extension. Extension is a quality abstracted from an extended object, just as "man" is abstracted from "Peter". One planet has a certain length of extension, just as Peter has a certain amount of "man". Another planet has another amount of extension, just as Paul has another amount "man". It is wrong to say that Peter is in "man"; it is equally wrong to say that a planet is in extension. The "in" relation is not one with space as a quality of an object (body).

If that were the case, space would be derived from the object by abstraction. The relation of space to an object in it is such that no comparing of similar marks or qualities in two objects yields space. All such marks or characteristics of objects are derived from the objects. Space can not be derived from an object. After a body and all of its qualities have been taken away, space is left. The space occupied by an object is quite different from the extension which is possessed by that body. The body carries its extension with it, when it moves. But it moves from one part of space to another. It remains in space, but it does not carry space with it.[1.] Qualities or determinations are possessed by objects. A planet possesses heat, weight, or extension. Peter possesses colour, weight or "manness." But space does not refer to an object as a content. A planet does not possess space as a quality or determination. It is in space. Space is its form. The relation expressed by the preposition "in" is the form-content relation. The form of an object is not capable of being abstracted from the object, but the object is in the form. The planet is in space.

1.) Euler XV: see above p. 23.

The planet, as a particular, and all general concepts (which are its determinations) are in the form. Space is the form of intuition. It precedes logically an object and all of its qualities.

Kant had made two advances in his doctrine of the ideality of space. To explain the relation between space and a particular space, he had come to regard space as a pure intuition. To explain the relation between space and a particular object in space, he had come to regard space as a form of intuition. Knowledge about a particular extended object is not a direct knowledge, or intuition, of extension. Knowledge of any general concept, such as "extension", is gained only by abstracting qualities from particular objects. On the other hand, knowledge of a particular space is a direct knowledge of space. All of the qualities of space are in any particular space. Similarly, if all of the qualities be abstracted from an extended object, "extension", as one of those qualities, is abstracted. But if all of the qualities be abstracted from an object in space, space remains. Space is the form of intuition.

In this first formulation of the doctrines of space, Kant was not concerned with specific problems about the

nature of judgments in Mathematics, either pure or applied. He reached these doctrines only in answer to the questions: (1) What is the logical relation between "space" and "a particular space"? (2) What is the logical relation between "a particular space" and "an object in it"? These two questions were raised by the original position that space is a concept of the understanding. The only outside influence was Euler. He caused Kant to discover the singular relation, and the form-content relation. The singular relation between "space" and "a particular space" causes all truth about "this particular space" to be immediately truth about "space in general". The form-content relation between space and an object in it causes all truth about the object to be conditioned by the nature of space. Both relations are in opposition to such relations as that between extension and an extended object, or that between "man" and Peter.

In working out the relations between space, the general concept; space, the particular concept; and the object in space , Kant realized that knowledge (in Geometry and Mechanics) is a priori and synthetic. Yet this knowledge is of the highest certainty. He realized that the certainty of this

knowledge is further evidence of the nature of space.

Space is a pure intuition, not only because of its differences from other concepts, but also because of the certainty of truth in pure Geometry. Space is a form of intuition, not only because of its relation to an object in it, but also because of the certainty of truth in applied Geometry. On the other hand, a priori synthetic truth[1] in Geometry is explained by the nature of space. The nature of truth in Geometry is proof of the nature of space; the nature of space is an explanation of the nature of truth in Geometry. With the full realization of this relation between nature of truth in Geometry and his doctrines of space, Kant developed these doctrines in the Dissertation.

The distinction between the doctrines of space as stated in the Dissertation and the way in which Kant first formulated them in 1769 is overlooked by Vaihinger.[2] In spite of the accuracy of his account of the two [illegible] problems (of pure and applied Mathematics), Vaihinger fails to see that it was not in answer to those problems that Kant made his earliest formulation of the doctrines of space. Vaihinger's analysis of the doctrines in the Dissertation would be true of Kant's earlier thinking in 1769 only if, throughout that thinking, Kant had known that the judgments in Mathematics are a priori

1.) This line of thought must not be confused with the critical problem of a priori synthetic judgments, with which Kant was first concerned after 1770.
2.) Vaihinger I. pp. 327-334. [illegible]

and synthetic. Vaihinger says, (II. 434.)

> "Die synthetisch-apriorische Natur der Urtheile der reinen Mathematik erforderte, dass der Raum als reine Anschauung gefasst werde (Diss. Sec. 15, C.); dass derselbe auch als 'blosse Form' unserer Anschauung gelte, war durch die Natur der reinen Mathematik als solcher noch nicht nahe gelegt, sondern wurde erst durch die durchgaengige Gueltigkeit der angewandten Mathematik fuer alle Objecte gefordert (Diss. Sec. 15. E.)."

This was true of the doctrines of space as a pure intuition and as a form of intuition, only after Kant gained the knowledge that truth in Geometry (pure and applied) is synthetic and a priori. Before he realized this, he found no more evidence that space is a pure intuition in the nature of Geometry, than in the nature of two such particulars as a pair of human hands, or, for that matter, in the nature of any group of particulars, such as Peter, Paul, and John. As I have shown above [1.], it is the contrast between space, as a particular, and other particulars, such as Peter and Paul, which showed Kant for the first time, that space is a pure intuition. In the example of a pair of hands, the mind is forced in a particular way to make an intuition of space. This is not the case with usual particulars. But the evidence that space is a pure intuition is just as certain by the contrast with Peter and

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1.) See above pp. 25-28.

Paul, as by the contrast with a pair of hands or as in the nature of truth in Geometry. Either Kant discovered that truth of Geometry is a priori and synthetic before he knew that space is a pure intuition, or he discovered this after the formulation of that doctrine. In the former case, his development would have proceeded quite differently from the way which I have described above. For the fact that truth in Mathematics is synthetic and a priori was of revolutionary significance.[1.)]

If Kant had realized this fact before he formulated his doctrine of space as a pure intuition, the answer to the question of the possibility of such knowledge would have been more important than a general consideration of the nature of space as a concept of the understanding. He would have included any treatment of the concept of space in a discussion of the problem of a priori synthetic mathematical truth. Actually, he included the problem of the nature of truth in Mathematics in the section of the Dissertation[2.)] which was devoted to the nature of space.

In Diss. 15 A. and B., he ignored the problem of the

1.) Vaihinger, I. 269-275; see above p. 19.
2.) Diss. 15.

nature of truth in Geometry. In part of Diss. 15. C., he treated examples in nature as well as in pure Geometry. In Diss. 15 D., he used the nature of truth in Geometry to refute Leibniz, but, otherwise, he did not consider that truth. Only in Diss. 15 E. was he exclusively concerned with truth in Mathematics, and even then he was concerned only with its validity for nature. If Kant had begun in 1769 with the insight that truths in Geometry are a priori and synthetic, why did he use that fact only to reenforce doctrines of space which were, in every case, formulated in answer to more general problems? Why did he use the problem of a priori synthetic truth in Mathematics only as strong supporting evidence of doctrines which exist without that problem? The answer is that Kant's study of the Clarke-Leibniz Controversy led to the question of the nature of the concept of space, and not to that of the nature of truth in Mathematics. It is true that Kant had considered the problem of the nature of that truth after the paper of 1768, but only in a general form.[1.] The specific problem concerning the synthetic a priori nature of that truth confronted him later. In 1769, as I have shown, he did not turn first to the problem of a priori synthetic

1.) See above p. 2.

truth in Mathematics, in answer to which a doctrine of space would have to be worked out, but followed the problem of the nature of space closely. In the investigation of the nature of space as a concept of the understanding, he reached the doctrine of pure intuition, and the form of intuition. Then for the first time, he realized the specific nature of truth in Mathematics.

Vaihinger's view must, therefore, be conditioned. He says:[1.]

"Dass also das Problem der angewandten Mathematik bei der Wendung von 1770 mitwirkte, kann keine Frage mehr sein,--"

If by the "change of 1770", Vaihinger means here the entire development of the doctrines of space, as carried out by Kant in 1768-70, he is correct in holding that the problem of applied Mathematics[2.] assisted in that change. For it did, after the doctrines of space as a pure intuition, and a form of intuition had been formulated. If by the "change of 1770", Vaihinger means the change made by Kant in his doctrines of space upon his discovery that the judgments of Geometry and Mechanics are a priori and synthetic, he is correct. For upon that discovery, the already existing doc-

1.) II. 435.
2.) Vaihinger's view of the problem of pure Mathematics requires a parallel line of criticism.

trines of space were further developed in explanation of the nature of truth in pure and applied Mathematics. But if by the "change of 1770", Vaihinger means the change from the doctrine of space as an absolute reality to the doctrines of space as a pure intuition and a form of intuition, he is quite wrong. For these doctrines were developed, as I have shown, apart from specific problems concerning the nature of Mathematic After the development of the doctrines of space independent of the problems of the nature of truth in Mathematics, Kant realized for the first time that [illegible] truth in [illegible] that science is a priori and synthetic, and used his doctrines of space in explanation of this fact. With the supporting evidence of the nature of truth in Geometry and Mechanics, the doctrines of space were developed in the Dissertation, as Vaihinger says. But Kant did not arrive at them first in answer to the problem of an a priori synthetic knowledge in those sciences. I have directed my criticism upon Vaihinger, because I follow him in regard to the nature of the doctrines of space in the Dissertation, after Kant knew that they are established not only in their own right, but also with the support of the nature of truth in pure and applied Geometry.

Riehl[1.] and Fischer[2.] hold that the main doctrine

1.) Riehl: I. p. 339.
2.) K. Fischer: Eng. Trans. pp. 30-31.

in the paper of 1768 was the view that space is an intuition. It is obvious from Kant's use in 1770 of the example of incongruent counterparts that, properly interpreted, this example is a basis for the doctrine that space is an intuition. But it is my view,[1] in agreement with Vaihinger and Kemp Smith, that in 1768 Kant's chief concern was to show that space is an absolute reality. In 1768, the fact that absolute space is "not an object of outer sensation,"[2] meant only that space is an absolute reality, in spite of the fact that it is not an object of sensation. Far from attaching significance to the fact that space is not an object of sensation, but possesses a reality "intuitable to inner sense",[3] Kant merely noted this as a difficulty in the doctrine that it is an absolute reality. Until the doctrine of space as an absolute reality had been rejected in 1769, he found no significance in the fact that space is intuitable. Perhaps the doctrines are not incompatible, but this did not occur to Kant in 1768, if it ever did.[4] In 1769, after he had discovered the inadequacy of Clarke's and Leibniz's points of view, Kant realized that space is a pure intuition, as I have tried above to indicate.

We can turn now to Kant's development of his doctrines of space as a pure intuition, and as a form of intuition, with

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1.) See above p. 1; Vaihinger, II. 525; Kemp Smith, p. 163.
2.) Handyside, p. 28.
3.) Ibid.
4.) Vaihinger, II. 310.

the supporting evidence of the nature of truth in Geometry. Kant discovered that truths in Geometry are a priori rather than a posteriori, and synthetic rather than analytic. What are the meanings of these distinctions? In accord with my above criticism of Vaihinger, we must be careful not to treat these distinctions as containing a fixed meaning for Kant before his investigation of the nature of space. He did not recognize before 1769 that truth in Geometry and Mechanics is either synthetic or analytic, a priori or a posteriori and set out to discover which it is. The opposite was rather the case. His investigation of the nature of the concept of space proved to be so fruitful that he was forced to seek a terminology which would express the meaning of his results. His discovery that truth in Geometry and Mechanics is a priori and synthetic was not the light of 1769. It was three steps removed from that light. His first step was to investigate the concept of space as a concept of the understanding. His second step was the discovery that space is a pure intuition and a pure form of intuition. His third step was the realization that these doctrines are of particular significance in reference to the nature of truth in Geometry and Mechanics. The culmination of these advances was his description of truth in these sciences as a priori and synthetic.

The difference between "a priority" and "a posteriority" is a difference between two types of validity or truth. Truth flowing from the concept "man" is analytic. It is based upon the nature of particular men. Such truth is limited to the extent of

experience. Truth flowing, on the other hand, from the concept "space" is a priori carrying with it a complete necessity and general validity.[1] Truth in Geometry and Mechanics is the clearest example of this sort. The difference between analysis and synthesis is a difference between two types of mental activity.[2] The universal "man" is related to the particulars "Peter" and "Paul" by a comparison of marks or characteristics. The universal is contained, at least implicitly, in (that is, under) the concept "man". The universal "space" is related, however, to a particular in a different way. "This space" is space. Whether space is constructed[3] or contemplated by[4] the mind, it is not obtained by analysis. In the first case, it is synthetic because it is obtained by a procedure which is the direct opposite of analysis. In the second case, it is synthetic only in the negative, neutral sense that it is not the result of any activity of the mind, and is, therefore, non-analytic.

This was Kant's discovery in 1769 that mathematical truth is a priori and synthetic. How is such truth possible? His original doctrine of space as a pure intuition warranted this question in regard to all truths about space, no less than in regard to those more specifically accurate ones about it, which

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1.) B. p. 4. "Notwendigkeit und strenge Allgemeinheit sind also sichere Kennzeichen einer Erkenntnis a priori, und gehoeren unzutrennlich zueinander." The problem whether a priority involves a subjective necessity or a necessity for objects is considered below pp. 68-81.

2.) Vaihinger, I. 269-275, see above p. 19n. Vaihinger claims that the distinction between analytic and synthetic judgments was originally the same for Kant as the distinction between logical and real judgments, that in 1768-70 a difference between these distinctions emerged, which was lost again later. (I. p. 332 n.)

3.) Diss. 15 cor. (end.) and the example of incongruent counterparts

4.) Rest of Diss. 15.

such men as Euclid and Newton had formulated. In 1769, Kant ended, however, his investigation of the concept of space, and turned his attention to the nature of truth in Geometry and Mechanics. Such truth is a priori because it is not limited to the number of particulars observed in experience. Truth in Geometry about triangle ABC is a priori, because it is ipso facto truth about any triangle. Truth in Mechanics is similar. Judgments in Geometry and Mechanics are, furthermore, synthetic. They do not concern a particular object, such as "gold", whose concept is contained under a general concept "metal". The concept in the logical predicate is neither a general concept, such as "metal", under which the logical subject is included, nor is it a specific quality, such as "yellow", of the logical subject. The logical subject and predicate are independent of each other. Judgments in pure Geometry and Mechanics deal with space. The objects with which judgments in applied Geometry and Mechanics deal are in space. Space is a form of the intuition of those objects. That judgments in pure Geometry and Mechanics should, before experience, be true, and that judgments in applied Geometry and Mechanics should be true for the objects of nature, is explicable only if space is a pure intuition, and a form of intuition. 1.)

In Diss. 15. C., Kant used the nature of truth

1.) Vaihinger, II. 434.

in Geometry as evidence of the doctrine of space as a pure intuition. He also used the example of incongruent counterparts as specific evidence of the nature of propositions in Geometry. He also reversed the procedure,[1.] and set the doctrine of space as a form of intuition as the explanation of the validity of judgments in Geometry in respect to the objects of nature. Thus, in Diss. 15. we do not have the doctrines of space as Kant first developed them in 1769, but a more thorough statement of them designed especially to explain the nature of the truth of propositions in Geometry and Mechanics, both as true propositions, and as true propositions in respect to the objects of nature. Such an explanation was necessary in view of the discovery that these truths are synthetic and a priori. This discovery was made after the earlier treatment of the concept of space as a concept of the understanding.

1.) In Diss. 15. C. and 15. E.

In the first paragraph of the Corollary to Diss. 15., Kant developed his doctrine of space as a pure intuition more explicitly. He said that the relation between the parts of space and space is different from the usual part-whole relation. In the latter relation, the parts contain the ground for the possibility of the whole. Peter, for example, is tall; but Paul is short. The characteristic "tallness" is a constituent part of Peter. Peter is determined by that constituent part. He is likewise determined by all other constituent parts. They are necessary to the existence of Peter. They make Peter what he is. They precede Peter, not temporally, but in the sense of containing the conditions under which Peter is Peter. In this type of relation, the constituent part "tallness" precedes the whole "Peter" and is the ground for its possibility. The parts precede the whole. This -- said Kant -- is not the sort of part-whole relation of various spaces to space. Particular spaces are not (in the strict sense of the word) parts of the whole,

but merely limits in the whole.[1] They are not parts in the whole as the constituent part, "tallness" is a part of Peter. There is no such thing as a long space or a short space, as there is a tall or a short man. "Length" or "shortness" refer to things in space; not to space. The "thisness" and "thatness" of particular spaces are not qualities of space.[2] Space is an infinite given whole. Only if infinite space be given can any particular space be marked out in it.[3]

In the last paragraph of the Corollary to Diss. 15., Kant raised a question independently of the doctrines of space as a pure intuition, and as a form of intuition:

1.) Diss. 15. C.n.
2.) See above pp. 25-27.
3.) Diss. 15 cor. First paragraph: " --- in quibus, non sicut leges rationis praecipiunt, partes et potissimum simplices continent rationem possibilitatis compositi, sed, secundum exemplar intuitus sensitivi, infinitum continet rationem partis cujusque cogitabilis ac tandem simplicis s. potius termini. Nam nonnisi dato infinito tam spatio, quam tempore, spatium et tempus quodlibet definitum limitando est assignabile, et tam punctum, quam momentum per se cogitari non possunt, sed non concipiuntur, nisi in dato jam spatio et tempore, tanquam horum termini."

namely, the question whether the concepts of space and time are innate or acquired. He admitted there that the question seems to have been answered already by his previous arguments. The doctrine that space is a pure intuition, is, in its truest significance, a direct answer to that question. Kant had discovered, with the help of Euler, that the concept of space is neither abstracted from sense, nor gained by reflection. "This space" had been shown to be the same as "space". Space is immediately known, and is a pure intuition. The a priori synthetic nature of truth in Geometry reenforces this fact. The whole of space precedes and determines the parts, which are limits in it. Infinite space is given, and contemplated in Geometry. Yet, in the face of these doctrines, Kant raised the question whether the concept of space is innate or acquired. He answered this question [1.] by saying that

1.) "Verum conceptus uterque procul dubio acquisitus est, non a sensu quidem objectorum, (sensatio enim materiam dat, non formam cognitionis humanae,) abstractus, sed ab ipsa mentis actione, secundum perpetuas leges sensa sua coordinante, quasi typus immutabilis ideoque intuitive cognoscendus."

space is acquired from the action of the mind on the occasion of experience. There is an activity of the mind from which the concept of space arises. This is a different doctrine from the rest of Diss. 15. with the exception, possibly, of the example of incongruent counterparts. These conflicting tendencies in Diss. 15. assume, as we shall see, greater proportions in the Critique.

Kant's doctrines of space in the Aes.[1.] are the outcome of his doctrine of space as a pure intuition as expressed throughout Diss. 15. with the exception of the end of the Corollary, where he spoke of the activity of the mind, and with the possible exception of the example of incongruent counterparts in 15. C., if that example be interpreted as showing that space is actively constructed by the mind.

1.) In my references to the Aesthetic, I shall use the abbreviation, Aes. Roman numerals refer to the numbered paragraphs under the heading "Von dem Raume". Unless the first or second edition is specifically referred to, as A or B, the reference is to either. I shall refer to the Analytic as Ana.

In every other part of the Diss., space is the pure intuition. It is contemplated, and not constructed. It is a presented or contemplated whole. In the Aesthetic, the example of incongruent counterparts is omitted. Its only significance in the Dissertation is from the point of view that the mind constructs space. In that example, the mind is forced to act or construct. It shows that there is an activity of the mind. But in the midst of the development of the doctrines of space as a contemplated whole it is out of place, and presents a point of view alien to that doctrine.

On the other hand, the opposite tendency of Kant's thought, which led to the position of parts of the Analytic,[1.] was also expressed in the

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1.) The Ana. contains some portions which reflect the earlier view of the Aes. For example, B. 123. "Erscheinungen wuerden nichtsdestoweniger unserer Anschauung Gegenstaende darbieten, denn die Anschauung bedarf der Funktionen des Denkens auf keine Weise."

Dissertation. The doctrine that space is acquired from an action of the mind, is quite different from the doctrine that it is contemplated in the science of Geometry. Space is constructed by an activity of the mind which is excited by outer sensation. Space is determined by that activity of mind. This is a different answer to Euler's question. Space is not abstracted from the objects of sense, but it is abstracted. It is gained by the mind in a certain way. It is not presented or contemplated. From the second view in the Diss. grew the doctrine in the Ana.[1] that space is determined by a synthetic activity which begins with the manifold of sense. We shall turn now to the developments of Kant's doctrines of space after 1770. We shall find that he developed the two doctrines of space more fully, but that he failed to make them compatible with each other. In the light of their incompatibility, we shall find[2] that he combined

1.) A 162-163; B 203-204; see below pp. 130-131.
2.) See below pp. 139-141.

them in the Dissertation only on the assumption of a thoroughgoing dualism between sense and intelligence.

PART II.

In his original formulation in 1769 of the doctrines of a pure intuition and a form of intuition, Kant was concerned solely with an analysis of the concept of space in contrast to the usual general concepts. Reenforcing these doctrines in 1770 was the fact that there is a priori synthetic truth in Geometry and Mechanics. In nearly every portion of Diss. 15., Kant gave the doctrines of space a greater significance than in his original formulation of them. Prior to sense-experience, the mind contemplates the whole of space in Geometry, and describes a priori and synthetically various relations in space. Space is not, therefore, a potential form, an unconscious mental endowment or tool, used by the mind in Geometry and Mechanics. If that were all that it is, then its difference from the usual general concepts, such as "man" or "extension", would be overshadowed by its likeness to them. Both would be unconscious mental elements, or functions, used by the mind at various times in various ways. "Space" and "extension" would be

different types of logical elements, but both would be logical elements, and nothing more.

Such a doctrine was more in line with Kant's original argument in 1769. It is the only one which is compatible with his most mature teaching in the Ana.[1.] But in Diss. 15. and in the Aes., he stated another doctrine. The whole of space is contemplated prior to sense-experience. Space is presented in its entirety <u>a priori</u> as genuinely as a colour is seen <u>a posteriori.</u>[2.] In pure Geometry and Mechanics, space, a conscious representation, is described. In Diss. 15., this doctrine is preponderant. In parts of the Aes., this doctrine is obscured (especially in the B. Edition) because of Kant's attempt to separate his argument into two different expositions. He set out to show in the Aes. by two independent arguments that space is the contemplated infinite whole, with which the mind is concerned in Geometry and in sense-experience. In the first of these arguments, (called in Edition B., a metaphysical exposition) he proceeded with a so-called assumptionless

1.) See below pp. 131-133.
2.) Kemp Smith p. 92.

investigation of the representation of space; in the second (called in B., a transcendental exposition) he proceeded upon the assumption that there is a priori synthetic truth in Geometry, and used this fact as evidence for the nature of space. There is some likeness between these two procedures in the Aes., and the two stages in Kant's thinking in 1769, but there is an important difference. In 1769, Kant undertook an investigation of the concept of space, as a concept of the understanding, with no idea of where it would lead him. The one fact upon which he based his investigation was that space is neither absolute (as Clarke had claimed), nor relative (as Leibniz had claimed). The result was that his first doctrine of space in 1769 was a logical contrast, proving nothing more than that the nature of the concept of space failed to conform either to the nature of a particular, such as Peter or Paul, or to the nature of a general concept, such as "man". In the Aes. the situation was quite different. His most important assump-

tion there was that space could be discovered to have a certain nature whether the search began with the representation of space, as such, or whether it began with the assumption that in Geometry there is a priori synthetic truth. On these two different assumptions emerge one and the same doctrine of space. Space is contemplated in Geometry (and prior to sense experience) as an infinite given whole, that is, a conscious representation. This was what he wanted to show by the two procedures in the Aes. His original search in 1769 into the nature of the concept of space was, therefore, only superficially like his metaphysical exposition of space in the Aes. In the former he had shown only that the concept of space is in a logical contrast with general concepts, while in the latter he showed that an assumptionless investigation of the representation of space leads us to the same infinite

whole of space which we know to be the source of a priori synthetic truth in Geometry. In both the metaphysical and the transcendental expositions in the Aes. he showed that space is an infinite whole, a pure intuition. In neither of them did he show merely that the concept "space" is different from the concept "man" in the way that he had done originally in 1769.

It is unjustifiable to strip Kant's arguments in Aes. of their obvious meaning, and try to read into them merely his original doctrines of space of 1769, which he had formulated before he realized that truth in Geometry and Mechanics is a priori and synthetic. These original doctrines of space remain, even if the arguments in the Aes. be rejected, as they must be in the light of the teaching of the Ana. The doctrine of an activity of the mind (formulated in only two parts of the Diss. 15)[1] turns out, in the end[2] to be the seed out of which grew the doctrine of the Ana., which destroyed the Aes., and reduced Kant's doctrines of a pure intuition and a pure form of intuition nearer to the original significance of 1769. We must follow out in detail Kant's teaching

1). The example of incongruent counterparts and the end of the corollary.
2). See below, p.110-111.

in the Aes., and point out its relation to Diss. 15., before we follow his development of his doctrine of space in the Ana., and show its incompatibility with the Aes.

To understand the first two arguments of the Aes. and to show their relation to the Diss., it is necessary to analyze Diss. 15. A. more closely. Kant has two arguments there. First, in order that the materials of sense may be put into an outer sensation, space, into which those materials may be arranged, must be there. Second, granted those materials to be arranged in space, it is they, and not space, which affect the senses. These two arguments were designed to show the relation of space to outer sensation; but they do so by two different procedures. In the first case, Kant raised the problem, how can the materials of sensation be got into a certain form? In the second case, he asked, does an analysis of them yield the form, once they are complete in that form? His answer to these two questions is: To get them into a certain form, space must be prior to them; in an analysis of them, space must remain when they are abstracted.

In the Latin original of Diss. 15 A., [1] and in the

1). Hartenstein, III. p. 143.

German translation[1] there are three sentences; in the English[2] there are four. The second sentence in all of these renderings gives what I have called above the first argument, that is, the argument that space is prior to outer sensation as the latter occurs.

> "For I cannot conceive anything as located outside me unless I represent it as in a space different from the space in which I myself am; nor can I conceive things as outside one another unless I arrange them in different parts of space."

The first sentence of all these renderings, and all of the sentences after the second, give what I have called above the second argument, that is, the argument that an analysis of a sensation as a psychological fact does not yield space.

> "The concept of space is not abstracted from outer sensation." - - - "Therefore the possibility of outer perceptions, as such, presupposes, and does not create the concept of space. Inasmuch as, moreover, things which are in space - - - affect the senses, space itself - - - cannot be derived from the senses."

In the middle of Diss. 15 A., Kant treated the problem of the relation of space to a sensation as the latter occurs,

1). Meiner, Phil. Bib. 46B. p. 109.

2). Handyside, p. 59.

or comes into being. At the beginning and the end of that section, he was concerned with the problem of the analysis of a complete sensation, that is, with the problem set for him by Euler.[1.] In this way, he formulated the two arguments. In each of them, he showed the relation of space to sense-experience. His arrangement of them suggests that the first is the proof of the second.

In the first argument, Kant began with the assumption that an outer sensation is occurring, and set the problem: How is space related to this psychological process? In the second argument, he began with the assumption that an outer sensation in its entirety is there, and set the problem: How is space related to this psychological fact? The answers to both questions involve space, in the first case, as a framework, consciously contemplated inside of which a sensation may come into being; and, in the second case, as an essential ingredient of a complete sensation. In the first problem, Kant considered under what conditions sensation occurs. In the act of

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1.) See above p. 22 .

conceiving (or arranging) a thing outside of me, or of conceiving (or arranging) one thing outside of another, I must first conceive space in which I am and in which the objects are. Space is prior to outer sensation. It is temporarily prior to my act of arranging objects in a certain way. I am conscious of the way in which objects are to be arranged before I can arrange them in that way. If I conceive an object to be outside of me, I find that, first, I must conceive space in which I am, and in which that object is. In the second problem, Kant considered by what conditions a complete sensation with all of its component parts is determined. In a complete sensation, things in space affect the senses. From such a sensation, things can be abstracted or carved out. Yet, after their removal, space remains. In the first case, space was there before sensation (and will be there after it); in the second, space is there apart from objects and their qualities, which may be abstracted. In the one case, sensation is a psychological occurrence; in the other, a psychological fact. In both cases, space is a conscious representation. 1.)

1.) Compare A 20., B. 34. "Die Form (space) derselben (of appearances) aber muss zu ihnen insgesamt im Gemuete a priori bereitliegen, und daher abgesondert von aller Empfindung koennen betrachtet werden." Also A 42., B 60. "Jene (space and time) koennen wir allein a priori, d. i. vor aller wirklichen Wahrnehmungen erkennen, und sie heisst darum reine Anschauung:" Kemp Smith loc. cit.

Kant's arrangement of these two arguments is confusing. Needless to say, they are related to each other. Each shows in its own way that space is non-empirieal, but one may not be regarded as the proof of the other. Space is not prior to outer sensation because it remains if the objects and qualities of a completed sensation are abstracted. Abstracting from a complete sensation is not the same as the beginning or the end of that sensation. If space is prior to a sensation, it is also posterior to it. But neither its priority nor its posteriority involves the abstraction of objects and their qualities from sensation. In such an abstraction, a sensation is treated as complete, without reference to its coming into being or passing out of being. In both cases, sensation is psychological; but in the one, it is treated as it occurs, and in the other, as it is. It is unfortunate that Kant allowed the two problems to be mixed in Diss. 15. A., by stating one as the thesis and the other as the body of the argument. The priority of space to outer sensation is not evidence that space cannot be abstracted from sensation. Probably his adoption of the procedure of thesis and proof helped to hide this fact from him.

The existence of the two doctrines in Diss. 15. A. has been overlooked, however, by Kant's commentators.[1]

1). Vaihinger, II, 157, 160, 189-5; Kemp Smith, 99n; 103-4; Caird, I p. 286-288.

for a different reason. The obvious connection between Diss. 15 A. and Aes. I., together with the fact that only the doctrine of the priority of space to sensation is in Aes. I., has led them to consider this argument as the only one in Diss. 15. A. They have overlooked the fact that the doctrine, in Aes. II., of the relative necessity of space [1.] for outer appearance is the same as the doctrine, in Diss. 15. A., that space remains if an object and all of its qualities are abstracted. This relation between Aes. II., and Diss. 15. A., becomes unmistakable when it is realized that the doctrine of the priority of space to sensation is not Kant's only argument in Diss. 15. A. Once this relation is noticed, it becomes clear that the arguments of Aes. I., and Aes. II. unite to show the relation of space to our sense-experience in exactly the same way that the two arguments in Diss. 15. A. unite to do so.

It is not necessary to deal at length with the relation between Diss. 15. A. and Aes. I. In Aes. I., Kant restated

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1.) Vaihinger, II. pp. 193-194. The doctrine of the absolute necessity of space is different. See below, pp. 74-76 .

the problem of the relation of space to a sensation as the latter comes into being. He dealt with it more adequately there than in Diss. 15. A.[1] The representation of space, he said, is prior to outer experience. In order that a thing may be represented as in a different part of space from that in which I am, or in order that one thing may be represented as outside of and next to another, the representation of space must already be presupposed (schon zum Grunde liegen).[2] This is a repetition of the first argument of Diss. 15. A., with one or two alterations of terminology. Kant introduced the phrase (schon zum Grunde liegen) to bring the doctrine into outward conformity with the doctrine which followed in Aes. II. Yet the problem involved is the same in 1781 as in 1770. How are the materials of sensation got into a certain form? What is the relation of the representation of space to a sensation as the latter

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1). An alteration of terminology in the Aes. is the change from "concept" to "representation" which includes both concepts and intuitions. See above, p. 24n.

2). The phrase "zum Grunde liegen", is used in both Aes. I and II. It means in the first case, "lying at the basis of outer experience as that experience comes into being, or occurs;" in the second case, it means "lying at the basis of a complete, analyzable outer appearance." It includes both arguments of Diss. 15. A. It means, on the one hand, "at the basis of my act of arranging objects in sensation;" and, on the other, "at the basis of all abstraction of objects from a complete sensation."

comes into being? There is a complete conscious representation of space which precedes the representation of object A as outside of me, or of object A as outside of object B.

Of the three sentences in Aes. II., the first is a restatement of the thesis of Aes. I., and Diss. 15. A. Space is a necessary representation a priori which lies at the basis of all external intuitions. As Vaihinger says,[1.] Kant meant in Aes. II by "representation a priori" what he meant in Aes. I., by "non-empirical concept." The second sentence in Aes. II contains the doctrine of the absolute necessity of space, which we shall consider below. The last sentence repeats essentially the second argument of Diss. 15. A. Granted the materials of sensation to be arranged in space, those materials affect the senses. Granted the complete sensation, space cannot be abstracted along with the object of sense. In the first argument of Diss. 15. A., and the argument of Aes. I., space is a prior existing framework in which a sensation in all of its aspects comes into being, or in which outer experience occurs. In the second argument of Diss. 15. A., and in this part of Aes. II., space is an essential ingredient of a complete sensation or of an outer appearance. Outer appearances (things in space) affect the senses. An external appearance with all of its determinations may be abstracted. Upon their removal space remains. In Aes.

1.) Vaihinger II. p. 197.

I., Kant said that space was there before outer experience (and will be there after it); in Aes. II., he said that it remains if the object and the qualities be abstracted from a complete external appearance. He took over both arguments from Diss. 15. A. with alterations of terminology, but essentially the same meaning. In all three sections, space is a conscious representation. Throughout these arguments Kant was dealing with sense-experience, first, as an occurrence, and second, as a fact.

Kant's argument in Aes. II., raises many problems independent of the question of its relation to the Diss. and to Aes. I. Vaihinger's [1.] distinction between absolute and relative (subjective and objective) necessity aids us considerably to understand Kant's argument, but that distinction leads us into further difficulties. The precise meanings of these two types of necessity can be made clear only by analyzing Aes. II., further.

> "Der Raum ist eine notwendige Vorstellung a priori, die allen äusseren Anschauungen zum Grunde leigt."

A priority [2.] involves strict necessity and universal validity. This being so, the word "necessary" is superfluous in the above sentence. If "universal validity" means validity for

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1.) Vaihinger II, pp. 193-195.
2.) See above p. 44n.

the intuiting subject, then the first clause of this sentence contains only the doctrine of the absolute necessity of space, and leaves the question open whether or not it is also necessary for objects. If "outer intuitions" involve necessarily objects (that is, outer appearances), then the second clause contains the doctrine of the relative necessity of space. If "outer intuitions" do not involve necessarily objects, the second clause contains only the doctrine of the absolute necessity of space. The first sentence contains, we may say, the doctrine of absolute necessity in any case. It contains also the doctrine of the necessity of space for objects, if "general validity" means validity for objects, or if "outer intuitions" involve objects. Let us proceed to the second sentence.

> "Man kann sich niemals eine Vorstellung davon machen, dass kein Raum sei, ob man sich gleich ganz wohl denken kann, dass keine Gegenstaende darin angetroffen werden."

Our problems multiply upon a consideration of this sentence. Much depends upon the meaning of the word "davon.". It can refer either to "outer intuitions", or to "space", or it can be a filling-in word, of which there are many in German. If it refers to "outer intuitions", the sentence reads as follows:

> "One can never make a representation of outer intuitions (in such a way) that there is no space, although one can easily think that no objects are encountered in it."

If outer intuitions involve necessarily objects, Kant was stating only the doctrine of the relative necessity of space. But this cannot be the case. In the last part of the sentence, he referred to the possibility that no objects are encountered. Outer intuitions do not necessarily involve objects. If there are cases in which outer intuitions necessarily involve objects, space is necessarily at the basis of those objects (outer appearances). This is the doctrine of relative necessity. In all outer intuitions, space is necessary, whether or not such intuitions ever encounter objects. We cannot intuite except in terms of space. Space is not at the basis of any and every cognitive state, but it is at the basis of any and every intuition.[1] In showing the a priority of space, Kant claimed its absolute necessity for every intuition independent of its relative necessity for objects. He was not claiming that we are unable to think except in terms of space, but that we are unable

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1). I omit the doctrine of outer and inner intuition which is not essential to this argument.

to intuite except in terms of it. In some cognitive states, (that is, in conceptions), we do not think in terms of space. In others, (that is, in intuitions), we do. Kemp Smith is, therefore, wrong, when he says:

> (p. 103.) "The ground upon which the whole argument is made to rest is the merely brute fact (asserted by Kant) of our incapacity to think except in terms of space."

Actually, the ground upon which the doctrine of the absolute necessity of space rests is our inability to <u>intuite</u> except in terms of space. By translating the word "davon", as referring to "outer intuitions", we see that in both the first and second sentences of Aes. II., Kant stated the doctrine of the absolute necessity of space for intuitions, and the relative necessity of space for objects.

By translating the word "davon" as referring to "space", we find that literally the second sentence reads:

> "One can never make a representation of space (in such a way) that there is no space."

By treating the word "davon" as a word which merely fills in, we find that the second sentence reads:

> "It is impossible to represent that there is no space."

In any of these three cases, Kant meant something other than

the doctrine of relative necessity. Nor did he mean that the representation of space is necessarily involved in all intuitions. He did not refer either to our representations of objects, or to any special class of representations, such as intuitions. He turned his attention, in accord with his metaphysical method of exposition, to the representation of space, as such. Space carries with it the quality of necessity. I cannot think of space as absent. Space cannot be represented contingently. If I represent a tulip as red, redness is contingent upon my sight. Remove the tulip, or give me green glasses, and there is no redness.[1.] On the other hand, space must be represented necessarily. It cannot be thought away. This is the doctrine of the absolute necessity of space.

We have now arrived at two interpretations of the doctrine of absolute necessity; the one depending upon the translation of the term "davon" as referring to "outer intuitions"; the other depending upon the translation of it as referring to "space", or merely as a word which fills in. Which of these doctrines was Kant teaching in Aes. II.? Did he mean that space is necessary in all intuitive thinking, or that it is itself a necessary representation? The first of these

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1.) Cf. Euler; above p. 23.

doctrines turns out to include the second, if they are interpreted exactly. Closely interpreted, the representation of space is one case of intuitive thinking. According to the first interpretation of the doctrine of absolute necessity, space is involved in all intuitive thinking. According to the second interpretation of the doctrine, space is a necessary representation. In any representation of space, it is represented necessarily. Any representation of space is, however, a case of intuitive thinking. The necessity of space, as such, is hence one case of its necessity in intuitive thinking. These two types of necessity become even more closely identified in B. III. where any particular space is shown to be "space". This being so, the representation of space is not merely one case of intuitive thinking. All intuitive thinking is a representation of space. This ultimate identification[1.] of space, the necessary ingredient in all intuitive thinking, with space, the pure intuition, depends upon the intuitive quality of space and not upon its a priority. Pending the establishment of the former, the representation of space may be considered as only one case of intuitive thinking. The representation of space and the representation of x

1.) Compare Diss. 12; A. 20, B 34.; B. 376-377.

intuited triangles are $\underline{x} + 1$ cases of intuiting thinking. In the light of the teaching of B. III., these $\underline{x} + 1$ cases remain, but the representations of the $\underline{x}$ triangles are each representations of space. The necessity of the representation of space, as such, becomes identical with its necessity in all intuitive thinking.

Taken in the light of Kant's arguments in Aes. II., the doctrine of the absolute necessity of space is out of place. Without it, the argument in Aes. I., and the argument of relative necessity in Aes. II., unite, as they did in Diss. 15. A., to show that space is non-empirical. Considered, however, in a wider perspective, the doctrine of absolute necessity is important to Kant's teaching in the Aes., and gives us an important clue to the difference between that teaching, and, on the one hand, the original doctrines in 1769 of space as a pure intuition and, on the other, the teaching of the Ana. If we take Kant at his own word at the beginning of the metaphysical exposition, he was carrying out an assumptionless investigation of the representation of space. If he had followed that procedure closely, and if we were not allowed to interpret anything in that exposition except in terms of that procedure, then Kemp Smith would be right,

and the doctrine of absolute necessity would be based on Kant's blunt claim that we can not think except in terms of space. If we remember, however, as I pointed out above, that there is a difference between the metaphysical exposition of 1781 and the truly assumptionless investigation of the concept of space, such as Kant had undertaken in 1769, we gain much light about the doctrine of the absolute necessity of space. In the metaphysical exposition, Kant claimed to make an assumptionless investigation of the representation of space, but actually he set out to follow one of the two procedures, both of which were designed to lead to the doctrine of space as a pure intuition, contemplated in its entirety prior to sense-experience, for example, in Geometry and Mechanics. In the metaphysical exposition, he made no reference to the fact that there is a priori synthetic truth in Geometry, but throughout that exposition (no less than in the transcendental exposition) he was referring to our thinking in any case in which we contemplate the whole of space, for example, in Geometry, pure and applied. The sum of these cases is not all of our thinking, but it is all of our intuitive thinking. In the doctrine of the absolute necessity of space, he meant that in some of our thinking (in intuitions) we contemplate space; that is, we represent it as present. In such thinking, we can not represent space as absent. We can not think it away. This is true, whether or not in such thinking we are concerned with objects (outer appearances). In all of

our thinking, we do not contemplate space. The doctrine of absolute necessity refers only to all of our thinking in which we contemplate space (viz. intuition).

There is a parallel between the role of space in Geometry, as an example of the doctrine of absolute necessity, and its role in sense-experience, which is the doctrine of relative necessity. In Geometry, space is contemplated, and is necessarily there, whether or not objects are contemplated. In sense-experience, outer objects are contemplated, and space is necessarily there at the basis of them. In Geometry and in sense-experience we cannot think except in terms of space. The mind can not follow out the axioms and propositions of Euclid without thinking in terms of space. The mind can not represent outer objects without thinking in terms of space. In Geometry and in other cases prior to sense-experience, space lies in the mind as a complete conscious representation. Geometry is an example of a science in which it is contemplated. In this and other cases it is contemplated prior to sense-experience. Kant's earlier teaching (in 1769) was quite different from this, and his later teaching (in the Ana.) contradicts it.

The problem of the relation of the doctrine of absolute necessity to the doctrine of relative necessity raises many difficulties. Did Kant mean by the doctrine of absolute necessity that space is a necessary representation (for example, in pure Geometry) whether or not objects are encountered in it, and, in some cases (for example, in applied Geometry) objects <u>are</u>

encountered in it? Did he mean that space is necessary in all intuitive thinking and that objects are encountered in some of this intuitive thinking (as in applied Geometry and experience) while in other intuitive thinking, (as in pure Geometry) it is not? If Kant meant this, his argument in Aes. II. runs as follows: Space is a necessary representation. In some cases, objects are encountered in representing it. Space is, therefore, at the basis of objects. Stated differently, but with essentially the same meaning, his argument would run as follows: Space is necessary for all intuitive thinking. Some intuitive thinking encounters objects. Space is, therefore, necessary for that intuitive thinking which encounters objects, and it is at the basis of those objects. There is much evidence that Kant meant this in Aes. II. "A priority" means general validity, and this may mean, as we have seen, validity for objects. Also the term "outer intuitions" in the first sentence may refer to objects. Kant may mean in the second sentence that objects are encountered in the representation of space, and hence space is at the basis of those objects. That is, in all intuitive thinking, space cannot be represented as absent whether or not an object is encountered (and, in some cases, objects are encountered). If the first two sentences are interpreted in this way, Kant included the doctrine of relative necessity within the doctrine of absolute necessity, and it is quite

justifiable that he should proceed in the last sentence to formulate the doctrine of relative necessity more exactly.

> "Er wird also die Bedingung der Moeglichkeit der Erscheinungen, und nicht als eine von ihnen abhaengende Bestimmungen angesehen, und ist eine Vorstellung a priori, die notwendigerweise auesseren Erscheinungen zum Grunde liegt."

The word "therefore" at the beginning of this sentence is further evidence that Kant was drawing the doctrine of relative necessity as a conclusion from the preceding argument. The conclusion is the same as the premiss stated in the first sentence with the word "intuition" replaced by the word "appearances." The term "appearances" is equivalent to objects.[1] In the concluding sentence to Aes. II., Kant formulated both types of necessity.

However unsatisfactory it may be, I am forced to adopt another interpretation of Aes. II. Strictly stated, the doctrine of absolute necessity does not include the doctrine of relative necessity and yield it as a corollary.[2] The latter doctrine follows from the former only on the assumption that, in some cases, objects are encountered in the representation of space. The one follows from the other only on the

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1.) I make no reference to the problem of the relation between "appearances" and "objects".
2.) Compare Vaihinger II, pp. 193-194.
3.) German: "also."

assumption that some intuitive thinking encounters objects. Strictly stated (as in the second sentence of Aes.II.), the doctrine of absolute necessity means that the representation of space is necessary whether or not objects are ever encountered in it. Space is necessary in all intuitive thinking, whether or not that thinking ever encounters objects. Perhaps it never does. In that case, the doctrine of relative necessity means that space underlies objects (outer appearances), and the doctrine of absolute necessity means that space is a necessary representation in spite of the fact that objects are never encountered in it. In that case, the doctrine of relative necessity means that space underlies objects (outer appearances), and the doctrine of absolute necessity means that space is necessary in all intuitive thinking in spite of the fact that objects are never encountered in such thinking. Perhaps the representation of space in pure Geometry is never a representation of objects in nature. Perhaps intuitive thinking in pure Geometry never involves the objects of nature. In any case -- says Kant in the second sentence in Aes. II. -- the representation of space is necessary. It is at the basis of all intuitive thinking. The doctrine of relative necessity does not follow ipso facto from the doctrine of absolute necessity. To assume that the

former follows ipso facto from the latter is to assume that the subjective conditions of representations in general include the conditions of the representations of objects. This cannot be assumed.[1] Considered without reference either to the first or the third sentence of Aes. II., the doctrine of absolute necessity is stated in this way in the second:

> "--ob man sich gleich ganz wohl denken kann,
> dass keine Gegenstaende darin angetroffen werden."

In this clause Kant did not say that in some cases, objects are concerned. He meant that we can easily think that objects are never encountered in the representation of space. Further evidence for this view lies in the fact that Kant took over the doctrine of relative necessity from Diss. 15.A., whereas the doctrine of absolute necessity was not in the Diss. Further evidence lies in the fact that Kant treated pure and applied Geometry as separate, and did not maintain that the former includes the latter.

I have mentioned the advantages accruing from the point of view that the doctrine of absolute necessity includes that of relative necessity as a corollary, and pointed out much evidence for this interpretation. I have proceeded to show why I think the opposite interpretation is correct. I do not claim that this interpretation is satisfactory in every respect. The word "hence" at the beginning of the last sentence is certainly evidence that Kant was arguing from

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1.) Compare Diss. 26, the first formula.

premiss to conclusion. In the second sentence there is, however, a doctrine of absolute necessity from which relative necessity cannot be derived. However unsatisfactory the conclusion may be, we must say that Kant put two independent arguments in Aes. II., only one of which came from the Diss. One result of this view of the doctrine of relative necessity as a second main argument, is that the connection of the argument with Diss. 15. A. assumes a greater importance. Vaihinger[1.] and Kemp Smith[1.] are not justified in holding that there is no connection between Aes. II. and the Diss.

The problem of the relation of the doctrine of absolute necessity to the doctrine of relative necessity involves the problem of the relation of pure Geometry to applied Geometry, and of both of them to experience. Adickes says that in the completed theory of the ideality of space, there was but one problem for Kant -- namely, how is Mathematics possible?

> "----Man darf also weder mit Paulsen behaupten, dass Kant nur das Problem der angewandten Mathematik eroertert, noch mit Fischer, dass es nur das Problem der reinen ist, noch mit Vaihinger, dass es beide sind (bald vermischt bald---von einander getrennt): man muss vielmehr sagen, dass Kant nur ein Problem kennt, die moeglichkeit der Mathematik ueberhaupt." (128 n.)

1.) Vaihinger, II., p. 184-185; Kemp Smith, 103-104.

We are not yet concerned with the role to which Kant relegated pure Geometry in the Ana. We have seen that in the Diss., he treated the two types of Geometry separately. In Aes. II., due to his metaphysical method of exposition, he made no reference to either type of Geometry. But his doctrines lead to important results in regard to the nature of the relation between the two types of that science. From the doctrine of absolute necessity it cannot be assumed that objects are necessarily involved in the representation of space. Similarly, from the nature of pure Geometry, it cannot be assumed that the objects in that science are ever objects in nature. To assume that judgments about intuited triangle ABC ever concern an empirical, intuited, three-cornered object, is to make the same mistake as to argue from the doctrine of absolute necessity to the doctrine of relative necessity. In both cases, the false assumption is made that the subjective conditions of representations in general include the objective conditions of representations of objects. If in Aes. II., Kant had argued from one type of necessity to the other, as Vaihinger says that he did, this would be ground for assuming with Adickes that Kant

identified pure and applied Geometry. But this is not the case. He put both types of necessity into Aes. II. He derived one of them from Diss. 15. A. He did not unify pure and applied Geometry. Perhaps the objects of nature never conform to the descriptions in pure Geometry. The doctrine of absolute necessity means, with reference to Geometry, that space is necessary in pure Geometry whether or not there is a science of applied Geometry. Space is necessary to intuited triangle ABC whether or not that triangle ever coincides with an empirical, three-cornered object. Similarly, the doctrine of relative necessity means that space is necessarily at the basis of the objects of nature, whether or not there are only such objects in nature as those which are contingent upon the nature of our senses (as a tulip), or whether there are also objects in nature (as an empirical three-cornered object) which conform to the intuitions in pure Geometry. Because of the independence of the two types of necessity, we must assume that pure and applied Geometry are different.[1.)]

Before proceeding to the discussion of Kant's arguments in Aes. A. IV., B. III., let us sum up the results of our

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1.) I have argued in a circular fashion here, claiming that the independence of pure and applied Geometry is evidence for the independence of the two types of necessity, and vice-versa.

treatment of the first two arguments in the Aes. and their relation to Diss. 15. A. In all three sections, Kant formulated a psychological doctrine. Space is a conscious representation, prior to the coming into being of a sensation, and essential to a complete sensation. Space lies at the basis of external experiences, as they occur. It lies at the basis of external appearances, essential to them as they are. The representation of space is at the basis of external experience and external appearances. These two arguments interlock with each other in Diss. 15.A., are taken over into the Aes., and interlock with each other there. They exist in this peculiar relation to each other in the Aes., apart from the additional argument in Aes. II., that space is itself a necessary representation. The doctrine of absolute necessity is an addition to the two arguments of Aes. I., and II., which Kant took over from Diss. 15. A. It concerns our inability to intuite except in terms of space. The doctrine of the psychological priority of space to our sense experience (Aes. I.), the doctrine of the absolute necessity, and the doctrine of the relative necessity of space for outer appearances (Aes. II.) are integral portions of the doctrine that space is a pure intuition. We shall see this psychological doctrine to be in contrast to Kant's later view in the Analytic.

I shall now proceed to the Kant's development in Aes. A. IV., B. III. of the doctrine of space as an intuition. He formed his argument from Diss. 15. B In 1770, he had used the contrast between "space" and general concepts to show that the former is a singular representation. In Aes. A. IV., B. III., he, however, used the same contrast to show that space is non-conceptual, that is, an intuition. In the first part of that section he repeated the argument of Diss. 15 B. Particular spaces are included in space, and not under space, as Peter and Paul are included under "man". He made two changes in the argument of Diss. 15. B. He omitted the doctrine of the boundlessness of space. He made a contrast between the usual part-whole relation and the relation of the parts (or limits) of space to the whole of space. 1).

In Diss. 15. B., he had said that particular spaces are parts of the same boundless space; hence, "this space" is in space, and not under it, as Peter is under "man". In A. IV., B. III., he stated the argument more exactly. He said that "this space" and "that space" are parts (limits) of the same space, whether or not it be

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1.) Diss. 15. C.n, and 15 cor. (first paragraph).

boundless; hence, "this space" is in space, and not under it, as Peter is under "man". Whether "space" be merely another particular space greater than "this space", or whether it be an infinite, boundless space, its relation to "this space" is not like the relation of "man" to "Peter". In either case, the relation involved is different from the usual relation of general concept to particular. Kant did not raise the question whether space is infinite, or whether the concept of infinite space involves merely the possibility of a boundless progression in the intuition. Whether space be boundless or not, -- he said -- we cannot treat the concept "space" in the way that we treat the concept "man" in relation to "Peter." This argument is negative, just as the argument in Diss. 15. B. from which it was derived. Kant merely said what space is not.

Kant developed, however, his point of view further in A. IV., B. III by taking over the argument from the first paragraph of the Corollary to Diss. 15. There he had contrasted the relation of particular spaces to space with the usual part-whole relation. He had found that infinite space is given, and determines all finite spaces. These are limits in it. In A. IV., B. III, he repeated this argument with one modification. He treated the relation of the parts of

space to the whole, but he did not say whether the whole of space is infinite or not. The whole of space is prior to the parts. The parts are limits in the whole. A part is thought as "in" the whole.

> "Diese Teile koennen auch nicht vor dem einigen allbefassenden Raume gleichsam als dessen Bestandteile (daraus seine Zusammensetzung moeglich sei) vorhergehen, sondern nur <u>in</u> ihm gedacht werden."

Did Kant mean that the whole determines the parts? He did not say so explicitly. He did not say that the whole precedes the parts, or that they are possible only through it. He said only that they are thought as <u>in</u> it; that they are limitations of it. In the light of the opening paragraph of Diss. 15. cor.[1]. his meaning is clear. There he claimed that the "whole" of space is the infinite given whole with the parts preceded and determined by it. In Aes. A. IV., B. III, he omitted the question of the infinity of space, but he preserved in every other respect the doctrine of the opening paragraph of Diss. 15. cor. The parts of space are determined by the whole. The parts of space are beside one another in space. The preposition "in" means the dependence of "the included" upon the "including". The latter is the whole. He left undiscussed until the next section the

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1). See above, pp. 48-49.

question of the infinity of the whole. 1).

The argument of A. IV., B. III. is the fruit of Kant's thinking of 1769. It is an integral step in the doctrine that space is a pure intuition. It is a step forward from his argument in Aes. I. and II. In Aes.I. he had shown that space, as a conscious representation, is prior to outer experience. In Aes. II., he had shown that the same conscious representation is an essential ingredient of outer appearances. He showed in A. IV., B. III. that the same conscious representation is different in kind both from the qualities and the general concepts which make up the usual representations of objects. Space is different in kind from other conscious representations, such as "man" or "extension". Space is intuitive; man is conceptual. Space, as an intuition, is an unconscious form, or logical function, but it is also a complete, conscious representation. Space and "man" are different, as logical entities, but also as two different kinds of conscious representations. Space, as a representation, determines every representation of particular spaces. Man, as a conscious representation, is determined by conscious representations of particular men. This doctrine loses its significance if either "space" or

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1). Vaihinger holds differently, II. p. 222.

"man" be interpreted only as logical elements, as logical conditions of consciousness, and not as conscious representations. Kant gave such a logical treatment originally in 1769/, but in Diss. 15. and the Aes. he treated space as contemplated in Geometry and prior to sense-experience. (and returned to it in the Ana.) In A. IV. B., III, no less than in Aes. I., and II., he was stating the doctrine of a given whole of space.

The difficulty with the doctrine of Aes. A. IV., B. III. lies in reference to the argument which follows in the Analytic.[1] Can the intuition of space be determined by an activity of the understanding, and also be a whole which determines the parts? In A. IV., B. III. Kant developed the doctrine of the determining whole, and the determined parts without reference to an activity of the understanding. To summarize his arguments[2] in this section: (1). Space is a pure intuition because the relation of "this space" to "space" is not like that of "Peter" to "man". (2). Space is a pure intuition, because the relation of "this space" to "space" is not a part-whole relation, in which the parts determine the whole, but the part-whole relation of determined parts or limits and determining whole. Whether space be represented as an infinite (boundless) whole[3] or as the possibility of proceeding infinitely with the intuition,[4] the relation

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1.) See below, pp.110-111; Kemp Smith, pp. 94-97.
2.) I do not treat the last sentences of A. IV., B. III.
3.) Diss. 15. B., and 15. cor. first paragraph.
4.) Aes. A. V. after the first sentence.

of a particular space to "space" is neither the relation of a particular to a general concept, nor the relation of determining part to determined whole. It is the relation of determining whole and limits in that whole.

In the first sentence of Aes. A. V., B. IV., Kant said that space is represented as an infinite given whole. This can be taken to mean either the contemplated or the conceived infinity of space. It can mean either that space is presented and contemplated as infinite, or that it is only thought as infinite. It is represented as infinite either immediately or mediately. In line with his arguments in Diss. 15. C.[1] and the first paragraph in Diss. 15. cor.[2] Kant meant probably the presented or contemplated infinity of space. In the argument of Aes. A. V. that follows the first sentence, he took, however, the other view. There he said that the possibility of continuing without bounds[3] in the progression leads to the principle of infinity. This is not the doctrine of a contemplated whole of space. A principle of infinity is not a contemplated or presented infinite whole. If we arrive at a principle of infinity, we merely conceive the whole of space. The argument of A.V., after the leading sentence, is the doctrine of the conceived

1). "Intuitu, in quo contemplando scientia illa (namely, Geometry) versatur."

2). See above, p. 49n.

3). R.360. "Es kann uns kein quantum als infinitum gegeben sein, denn es wird nicht an sich selbst gegeben, sondern nur durch den progressus, der niemals als infinitus gegeben ist. Aber ein progressus in infinitum kann gegeben sein; indefinitum, dessen Grenze wir unbestimmt lassen."

or thought infinity of space. Kant treated the infinity of space as a boundlessness in the progression. He said that a general concept of space can determine nothing in reference to size or quantity.[1] Without boundlessness in the continuation of an intuition, there would be no concept of relations which would evolve a principle of infinity. In Diss. 15. B., he had contrasted the concept of space with the usual general concepts, such as "man" or "extension", and assumed that space is boundless. In Aes. A. IV., B. III., he had made the same contrast without assuming either that space is boundless or the opposite. In Aes. A. V. he considered the nature of the boundlessness of space. In doing this, he showed that no general concept of space, such as a "foot", or an "ell", can be formed. One size is no more "space" than another larger (or smaller) size. No particular space is "space", any more than any other particular space. There is always a larger space, which with equal justification may be considered "space". This boundlessness is essential to the attainment of a principle of infinity. Without it no concept of the infinite whole of space can be formed, and we have this concept of such a whole.

This argument is valid, but it contains two difficulties. In the first place, it does not show that space is non-conceptual. It is not a part of the doctrine of space as a pure

1.) Also, R. 354.

intuition. In the second place, in so far as it shows that space is only a conceived or thought infinite whole, it not only does not show that space is non-conceptual, but it even shows that space is conceptual. If we arrive at a principle of infinity, we represent the whole of space mediately.[1] If the infinite whole of space is conceived, then "space" and "man" are similar and not different. In other words, the very argument which Kant used in A. V. to show that space is an infinite given whole, turned out (as he saw after 1781) to show that it is conceived as infinite. It seemed to show that space is, after all, conceptual, with the parts determining the whole. The conclusions, furthermore, of the Analytic,[2] with which Kant had been concerned were in harmony with this. According to his later teaching, a synthetic activity of the understanding determines space, making the parts of space prior to the whole, and the whole determined by the parts. This doctrine, which threatened to destroy the arguments of the Aes., received support from the doctrine in A. V., of a principle of infinity.

For these reasons Kant recast his fourth argument on space in 1787. He italisized the word "given" in the leading

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2). See below, pp. 117-121; B. 202-3; Kemp Smith p. 94.

1). The problem of the identification of mediacy with conception, and immediacy with intuition is a further question.

sentence to show that he was not referring to a conceived infinity. He formulated a new argument which was designed primarily to show (parallel to B. III) that space is non-conceptual. To this extent, he was successful. His argument in B. IV., is an additional argument that space is non-conceptual. He wanted further, to show (even if in a negative way) that the whole of space is a given or contemplated infinite whole. In this he failed. His new argument is evidence that space is non-conceptual, but it is not evidence for the contemplated infinity of space. In so far as space is infinite, this argument shows that / it is a conceived infinite whole. Kant tried, however, to show more. Space is an infinite given whole --he argued -- in the sense that an infinite number of representations are in it, and not under it. Each particular space is represented immediately. In this sense, space is represented immediately as an infinite whole. An infinite immediately represented whole is one which includes an infinite number of immediately represented parts in it. Innumerable men exist under the general concept "man". There is, however, no general concept in which there are an infinite number of particulars, as an infinite number of particulars are in space. Kant used this argument parallel to the argument of Aes. B. III. It shows validly that space is non-conceptual, but

like A. V., it does not show that space is an immediately represented infinite whole. The number of representations in space is infinite, but neither space nor any of these representations can be treated as infinite without returning to the position of A. V. If space be infinite, it is represented mediately. Infinite space involves the possibility of continuing without bounds in the progression. Such infinity is conceived, and is evidence that space is conceptual. Similarly, if any one of the infinite number of representations in space be treated as boundless, it is represented mediately, which is evidence also that space is conceptual. In both A. V. and B. IV., Kant showed that we

have a conception of an infinite whole of space. We represent such a whole mediately. In A. V. he said that this conception is the result of an indefinite continuation in the progression of the intuition. In B. IV., he said that it is the result of an infinite number of particular representations in space. It is quite as invalid for him to assume an infinite contemplated whole in the latter case, as in the former. The number of representations in space is infinite, just as in A. V. The progression in the intuition is infinite. But each representation is finite. There is, in each case, a conception of the infinite whole of space.

Such a whole is not contemplated. In B. IV. Kant showed that space is intuitive. In this way B. IV. is an improvement upon A. V. As an attempt to demonstrate the leading sentence, that space is an infinite contemplated whole, one argument is as unsatisfactory as the other.

It will be remembered that in Kant's original treatment of the concept of space in 1769 he faced two separate but parallel problems. In the first place, he asked, what is the relation of a particular space to space in general? In the second place, he asked, what is the relation of space to an object in it? As a result of his investigation

of the first question, he discovered that space is a pure intuition; as a result of his investigation of the second, he found that space is a pure form of intuition. Upon his discovery that truth in Geometry and Mechanics is a priori and synthetic, he stated these two doctrines in Diss. 15., with this fact as supporting evidence. He stated the first doctrine in Diss. 15. C.; the second in Diss. 15. E.[1] In 1770 it was only in another section that he identified the pure intuition with the pure form of intuition.

In the Aes. it was also outside of the expositions of space[2] that he identified the pure intuition with the pure form of intuition. His division of his argument into the two expositions had, however, a curious effect upon the relation of the two doctrines of space to each other. In each exposition he was determined (from different premisses) to derive his entire doctrine of space. This meant that following either the metaphysical or the transcendental method, he intended to establish both the doctrine that space is a pure intuition, and that it is a pure form of intuition. The success of each of these methods, as well as the unity

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1.) Diss. 12.
2.) A. 20; B. 34-5.

of the structure of thought of which they form the two ingredients, depended, therefore, upon the validity of the identification of the two doctrines of space with each other. If the pure form of intuition is not also the pure intuition, it is quite possible that the assumptionless investigation of the representation of space might lead to the one, and not to the other. The transcendental investigation might lead also to the one and not to the other. These possibilities forced Kant, in identifying the pure form of intuition with the pure intuition, to make every characteristic of the one also a characteristic of the other. In the metaphysical exposition, he made no reference to the doctrine of a form of intuition, yet he derived all of the characteristics of the form of intuition and attributed them to the pure intuition. He claimed that space is a conscious representation prior to sense experience, and outer appearances. A pure intuition is, by definition, a conscious representation.[1.] He discovered the pure intuition at the basis of experience and appearances. In 1769, in his original formulation of the doctrine of the pure intuition he was not concerned with

1.) B. 377.

the problem of the relation of space to an object in it. He considered this problem separately. He faced it with the doctrine of pure intuition complete, and found it to yield the quite different doctrine of space as a pure form of intuition. Due to his identification of the two doctrines of space, and his belief that both could be derived by the metaphysical method, he discovered prior to and at the basis of objects, not the form of intuition, but the intuition itself. Had he remembered his definition of an intuition, he would have realized, as Erdmann says,[1.] that consciousness presupposes a synthesis of the sensuous manifold, and that a claim[2.] that a conscious representation is prior to and at the basis of sense-experience is a contradiction of that definition. A conscious representation cannot be prior to sense-experience, for, by definition, such a representation presupposes a synthesis of the objects of sense-

1.) See Erdmann, footnote to R. 357. "Ich erwaehne nur, dass Kant das Recht, diese allgemeine Form als Anschauung zu bestimmen, gegen seine eigene Fassung der Anschauung als bewusster Vorstellung in Anspruch nimmt, da alles Bewusstsein Synthesis des Mannigfaltigen voraussetzt."

2.) As in Aes. I. and II.

experience, which must be, therefore, prior to it. A conscious representation cannot be at the basis of the objects of sense experience, for, by definition, such a representation presupposes a synthesis, which begins with the objects of sense, which would require, in turn, another conscious representation at the basis of them, and so on, ad indefinitum. In other words, Kant's identification of the pure intuition and the pure form of intuition, led him in the metaphysical exposition to ascribe properties of the latter to the former. If he had realized that the only properties belonging genuinely to the former are qualities discoverable in the formal contrast of space and general concepts, he would have avoided the line of argument in Aes. I and II, which we have just traced. Similarly, in Aes. B. IV., he used the problem of the relation of space to an object-in-it not as evidence that space is a form of intuition, as he had done in 1769, but that it is intuitive.[1.] This argument breaks down, as we have found, when it is applied to an infinite space. Even in reference to finite space, its validity depends upon the unquestioned assumption that all

1.) Kant's identification of these two doctrines explains his assumption in Conclusion b that space is the form of intuition. Kemp Smith pp. 114-115.

representations are either intuitions or concepts. At its full value, it proves only that space is non-conceptual. In 1769, Kant had maintained that the relation of space to an object in it shows that space is a form; in B. IV., he claimed that it shows that space is intuitive. This change in his argument is a further result of his identification of the pure intuition with the pure form of intuition. He did not realize that this identification was leading him to an overstatement of the doctrine of space as a pure intuition. He tried to formulate throughout the metaphysical exposition the doctrine that a conscious representation of space is contemplated in its entirety prior to sense experience, that it is at the basis of outer appearances, and that it is in contrast to other representations, such as "man" or "extension." Whatever significance the doctrines of the Aes. retain, if interpreted as a treatment of the unconscious form of intuition, they contradict the doctrine of the pure intuition in the Ana., which presupposes a synthetic activity of the understanding beginning with the manifold of sense.

We have followed out the four arguments in the Aes., which, taken as a unity, show that space is a pure intuition. In the light of these arguments, we may understand better

Kant's thinking in 1769 when he turned from his study of Clarke and Leibniz to the consideration of the nature of the concept of space. His original doctrine that space is a pure intuition was the outcome of that study in 1769. He arrived at this doctrine without reference to the nature of truth in Geometry and Mechanics either as pure or applied sciences. He discovered later that truth in Geometry and Mechanics is synthetic and a priori, and, in the Diss., he formulated the doctrine of space as a pure intuition with this fact as supporting evidence. After 1770 (more clearly in the 2nd Edition of the Critique than in the 1st), he separated his arguments on space into those which deal directly with the representation of space (whether in Geometry or not) and those which presuppose the existence of a priori synthetic truth in Geometry. Both sets of arguments show that space is a pure intuition, and a pure form of intuition. There is no reference in either to an activity of the mind. Space is a pure intuition, pure, because it is prior to sense-experience, and intuitive, because of differences existing between it and all general concepts. In the four arguments in the metaphysical exposition of space as well as in the transcendental exposition in the

Aes., Kant treated space as a representation which (for example, in Geometry) is prior to sense-experience and is so real as a colour or a sound.[1] Space is at the basis of sense-experience, as a psychological occurrence. It is at the basis of an object of sense (outer appearance) as a psychological fact. It is a necessary representation. Its parts are preceded and determined by the whole. Its parts are <u>in</u> it, and not <u>under</u> it. These are Kant's arguments in the metaphysical exposition of space in the Aes. He analyzed the representation of space, in relation to experience, and as treated in Geometry, whatever be the nature of truth in that science. In the transcendental exposition of space, he used the nature of truth in that science as an explanation of the nature of space. The relation of the two expositions to each other, as well as the nature of the individual arguments in the former exposition, show that, contrary to his earliest doctrine of pure intuition in 1769, and to his later teaching in the Ana., he was concerned in the Aes. with space as a conscious representation.

1.) Kemp Smith, p. 92.

PART III

I have shown[1] that Kant put two different views of space in the Dissertation. According to the first of these, space is a pure intuition, contemplated in its entirety in Geometry. I have dealt with Kant's development of this doctrine after 1770 in the Aesthetic, and shown that throughout that section of the Critique, he formulated the view that space is a conscious representation prior to sense-experience. We shall turn now to the second of the views in the Diss., namely, that space is abstracted from an activity of the mind. We shall study Kant's development of this view in the Analytic. We shall examine his unsuccessful attempt to combine it with the view of the Aes. We shall discover how his teaching of the Ana. destroys the arguments of the Aes. In the light of his doctrines of space after 1770, we shall understand better the contrast between them as they appear in the Diss.

The second view of space in the Diss. is contained in two parts of Sec. 15. In the middle of 15. C., Kant put the example of incongruent counterparts. Two equal but incongruent hands (or triangles) are alike as far as internal arrangement is concerned, but they cannot be made to coincide. To explain their difference, the mind is forced to

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1.) See above pp. 51-54.

make a pure intuition of space, and to locate the hands (or triangles) in it. In this example,[1.] space is not presented or contemplated in Geometry, but constructed. The mind is constrained to act. There is an activity of the mind. This is neither an abstraction from the objects of sense, nor is it reflection. In this example, Kant returned to Euler's question: Is the concept of space abstracted from sense, or gained by reflection? Kant repeated[2.] that it is gained in neither way. But he did not say, as in other portions of Diss. 15., that space is presented or contemplated, and not abstracted at all. There is an activity of the mind through which the concept is gained. He repeated this view more fully at the end of Diss. 15. cor. His answer there to Euler's question is similar to the example of incongruent counterparts. Space is neither abstracted from sense, nor is it innate. It is gained by abstraction from an activity of the mind. Sensation excites this activity, but it does not contribute to the intuition. Space is a product of the constructive

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1.) See above pp. 30-32.
2.) See above pp. 21-24.

activity of the mind.

In this doctrine, and in the example of incongruent counterparts, Kant was at variance with the rest of Diss. 15. In the first argument of Diss. 15. A., he stated that the concept of space is prior to me, and to objects a and b, which I arrange outside of me, and outside of each other. There is an activity of the mind, but space is not abstracted from it. Space is prior to that activity. In the second argument of Diss. 15. A., he had stated that space remains when the elements of a complete sensation are abstracted. The concept of space is not acquired from an activity of the mind. In Diss. 15. B., he had made a contrast between space and the usual general concepts, such as "man", or "extension". He was not concerned with the manner in which the concept of space is gained, or arrived at by the mind, but with the nature of space, whatever the way in which it is represented. He contrasted "space" and "man", not two types of activity of the mind. In Diss. 15. C., he had said that space is contemplated. There is no activity of the mind. In Diss. 15. cor. (first paragraph), he had said that space is an infinite given whole, with the parts determined by it. In spite of these doctrines in other

paragraphs of Diss. 15., Kant said, in concluding that section, that the concept of space is abstracted from an action of mind.

Can his other doctrines of space in Diss. 15. be reconciled with this view? What is the nature of such a mental activity? There are only three possible types of mental activity from which space could be abstracted. The first type is a purely intellectual activity, having its origin apart from sense-experience, but yielding paradoxically a pure form of sense-experience, space. This is the view at the end of Diss. 15. cor. The second type is equally paradoxical, but in a different way. It has its origin neither in the intellect, nor in sense-experience. It yields space, as a pure intuition and a pure form of intuition, and begins apparently with sensitive[1.] parts or limits in it. This doctrine is the result of Kant's misunderstanding after 1770 of the nature of a synthetic activity of the mind.[2.] He failed at that time to realize that the only kind of activity which can yield space either as a pure intuition or as a pure form of intuition is an activity which begins with the objects of sense (sensuous manifold). He forgot that if there is a particular space with which a mental activity could begin, then to reach the whole,

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1.) Cf. Kant's distinction (Diss. 5.) between the matter and form of sensation, the latter being sensitive.

2.) See below pp.127-130; esp. p.130 n.

no such activity is necessary, for one particular space is "space" no less than a greater (or smaller) particular space. The third type of mental activity has its origin in sense-experience, from which it constructs space, as a pure intuition, and a pure form of intuition. Space is determined by an activity of the mind, which begins with the sensuous manifold. This is the view of the Analytic. It is in contradiction with the teaching of Diss. 15. A., and with the four arguments in the Aes.

Can space be prior to sensation, and be derived from activity of the mind? If so, such an activity must be completely intellectual, and must yield a sensitive whole. Can an activity of the mind yield the whole of space without having dealt already with the parts? Can an activity of the mind yield the whole of space unless the parts precede and determine the whole? If so, the activity is completely intellectual, in spite of the fact that the whole of space (including the parts) is entirely sensitive. The dualism of 1770 between sense and intelligence lies at the basis of Kant's reconciliation of the doctrine of the priority of

space to sense-experience with the doctrine of the activity of the mind from which space is abstracted. That dualism lies also at the basis of Kant's reconciliation of the doctrine that the whole of space precedes and determines the parts (limits) with the same doctrine of the activity of the mind. If that dualism be denied (as Kant denied it after 1770) then no activity of the mind can yield space unless that activity begin with the sensuous manifold, in which case sense-experience precedes space. If conception and intuition are not isolated but connected, there cannot be an activity of the understanding unless it begin with the sensuous manifold. Any activity which does not begin with sense-experience must be entirely intellectual, in which case, the dualism is retained; or it must begin with parts of space, in which case, space is not abstracted from it, but is already given before the activity begins. For any particular space is "space" as much as any other. If the dualism of 1770 be denied, then no activity of the mind can yield space unless that activity begin with sense-experience, in which case, the parts of space precede and determine the whole of space. If an activity of the understanding begin with the sensuous manifold, it can yield space only if space is not prior to but after that manifold. It can yield the whole of space only after having dealt

first with the parts. It cannot deal with the parts first, and then yield a whole which precedes and determines the parts.

A denial of the dualism of sense and intelligence, and the affirmation of a synthetic activity of the mind leads, therefore, to a direct contradiction of the teachings of the Aes. Space cannot be abstracted from an activity of the mind, and yet be prior to sense-experience. The whole of space cannot be abstracted from an activity of the mind, and yet precede the parts of space with which that activity has dealt. In the Diss., due to his dualism, Kant did not face these facts. He assumed that an entirely intellectual activity can yield an entirely sensitive whole, without the objects of sense being prior to space, and without that activity having dealt previously with the parts of space. Because of his dualism, he assumed that space is prior to the objects of sense; that the sensitive whole of space precedes and determines the parts; and that there is an activity of the mind from which space is abstracted. After 1770, he rejected the dualism between sense and intellect upon which this doctrine is based. We must now trace his rejection of the dualism and his formulation of

a theory of knowledge in which intuition and conception are bound together. In the light of this theory of knowledge, we shall see more clearly how he contradicted the teachings of the Aes. He retained them in the Critique either because he did not realize that he had contradicted them, or because, realizing it, he saw that the alteration required was too far-reaching to be undertaken.

It will be noticed as my argument progresses from this point that I differ in one respect from Kemp Smith in regard to the nature of the conflicting doctrines of space in the Aes. and Ana. respectively. I agree with him that there are these conflicting doctrines. He goes on, however, to maintain that Kant was unaware of them.

> (p. 96.) "But Kant does not himself recognize any conflict between this teaching (of the Ana.) and the doctrine of the Aesthetic."

Now I do not maintain the opposite. I do not claim that Kant was aware of these contradictions. I do claim, however, disjunctively that Kant was either unaware of them, or he was aware of them and realized that to attempt a reconciliation meant the destruction of the Aes. In the former case, certain passages (as at B. 160n.) represent a genuine attempt on his part to build up a unified doctrine of space in the

Critique, and Kemp Smith is not justified in claiming that Kant grafted one view upon the other.[1.] Only in the latter case may Kant be said to have been grafting. If the differences between the Ana. and the Aes. are as important as I believe them to be, Kant would have grafted the one upon the other only if he had recognized the conflict and had realized that such grafting was the least unsatisfactory solution. If he did not recognize the conflict, he cannot be said to have been grafting. Let us consider his teaching in the Ana.

> (B. 146.) "Sich einen Gegenstand denken, und einen Gegenstand erkennen, ist also nicht einerlei. Zum Erkenntnisse gehoeren naemlich zwei Stuecke: erstlich der Begriff, dadurch uberhaupt ein Gegenstand gedacht wird (die Kategorie) und zweitens die Anschauung, dadurch er gegeben wird; denn, koennte dem Begriffe eine korrespondierende Anschauung gar nicht gegeben werden, so waere er ein Gedanke der Form nach, aber ohne allen Gegenstand, und durch ihn keine Erkenntnis von irgendeinem Dinge moeglich;"

Thought -- according to Kant after 1770 -- is distinguished from knowledge, and the latter is bound inseparably with intuition. It is possible to think an object without knowing one. There is knowledge only when an intuition of the object corresponds to a concept of it. Only concepts are required for thinking, but both concepts and correspond-

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1.) Kemp Smith, loc. cit.

ing intuitions are required for knowledge.

> (B. 146.) "Sinnliche Anschauung ist entweder reine Anschauung (R. und Z.) oder empirische Anschauung desjenigen, was im Raum und der Zeit unmittelbar als wirklich, durch Empfindung, vorgestellt wird. Durch Bestimmung der ersteren koennen wir Erkenntnisse a priori von Gegenstaenden (in der Mathematik) bekommen, aber nur ihrer Form nach, als Erscheinungen; ob es Dinge geben koenne, die in dieser Form angeschaut werden muessen, bleibt doch dabei noch unausgemacht. Folglich sind alle mathematischen Begriffe fuer sich nicht Erkenntnisse;"

Sensible intuitions are of two kinds: First, the pure intuitions of space and time (as in Diss. 15. C.); second, the intuitions of the objects in space and time (as in Diss. 15. E.). Concepts corresponding to pure intuition give knowledge in form. Knowledge in form differs from thinking which is not knowledge in that its concept corresponds to a pure intuition, and is not, therefore, merely a concept with no content. It differs from the knowledge of the objects in space and time in that its intuition is pure, and not empirical. It is knowledge in form only. It does not tell us whether objects exist which could be intuited in such a form. The concepts of Mathematics do not give knowledge without a corresponding intuition any more than any other concepts. An intuition of objects in space is not, however, the only kind which can correspond to a concept of the mind. When a pure intuition corresponds to a

concept, there is knowledge in form only. When an empirical intuition corresponds to a concept, there is knowledge.

> (B. 147.) "Dinge im Raum und der Zeit werden aber nur gegeben, sofern sie Wahrnehmungen (mit Empfindung begleitete Vorstellungen) sind, mithin durch empirische Vorstellung. Folglich verschaffen die reinen Verstandesbegriffe, selbst wenn sie auf Anschauungen a priori (wie in der Math.) angewandt werden, nur sofern Erkenntnis, als diese, mithin auch die Verstandesbegriffe vermittelst ihrer, auf empirische Anschauungen angewandt werden koennen." 1.)

A concept in Mathematics becomes knowledge either by being applied to an empirical intuition, that is, to an empirical object in space, or 2.) by being applied <u>a priori</u> to an intuition (space or time) which could be applied to an empirical intuition. The latter is knowledge in form.

Kant refers to three different kinds of application. There is, first, the straightforward application of a concept to an empirical intuition. This means that the object is thought and intuited, and that the concept and intuition correspond to give knowledge. There is, second, the application <u>a priori</u> of a concept to an intuition. This occurs in pure Mathematics. In this type of application, the object is not necessarily existent. There is an object

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1.) Compare, B. 194-195.
2.) The second way is the significant addition.

which is thought and intuited, but since this object is not intuited empirically, there is no assurance that it exists. This is knowledge in form only. In both of these cases, the application depends upon a correspondence between a concept and an intuition. These two types of representations correspond. The concept is applied to the intuition. The third kind of application is quite different. It is the application of a pure intuition (space or time) to an empirical intuition. It is based upon the relation of a form of intuition to an empirical content. The pure intuition (space) is a form of every possible empirical intuition. If a concept has been applied to a pure intuition which is a form of every empirical intuition, it follows that the concept could be applied to any empirical intuition. If concept a has been applied to pure intuition b, and if pure intuition b could be applied to empirical intuition c, (since pure intuition b is a form of empirical intuition), then it follows that concept a could be applied (along with b) to c. Kant's statement of this doctrine is obscure, due to its inclusion in a clause (mithin auch die Verstandesbegriffe vermittelst ihrer) which breaks the train of thought. Yet it is vital to his argument. For only in the application

of a concept to an empirical intuition does knowledge occur. The significance of the doctrine is that it shows how we may, without gaining any knowledge (i.e., by means of knowledge in form), ascertain the conditions of knowledge.

Kant would seem, in a broad way, to argue thus: I am going to state a doctrine which, at first glance, will seem only to limit all knowledge to experience, and to deny that pure Mathematics belongs to experience, and, therefore, to deny that in pure Mathematics there is knowledge. If you follow me closely, however, you will see that in this doctrine something more important emerges than this negative conclusion. In pure Mathematics, without turning to experience, I find a way of accurately limiting knowledge, and of setting up conditions which knowledge cannot contradict. I do, in fact, deny that in pure Mathematics there is knowledge; but I hold that there is in it knowledge in form. This is no merely negative conclusion, for it is by means of knowledge in form that we can determine the possibility (limiting conditions) of knowledge. The application of a concept to an empirical intuition gives knowledge. The application of a concept to a pure intuition (which may

be applied to an empirical intuition) sets conditions under which knowledge occurs, and which knowledge cannot contradict. Space is a pure intuition, and a pure form of empirical intuition. In the application of a concept to the pure intuition of space, there is not knowledge, but only knowledge in form. This knowledge in form determines the conditions of knowledge. This does not mean that any intuition, which is not empirical, can determine the conditions of knowledge. There may be some intuition, which is not given to man. Unless such an intuition were also a form of empirical intuitions, it would not determine the possibility of them. The pure intuitions of space and time are not the source of knowledge, but they bear an important relation to it. They are intuitions to which concepts are applied, and which are the form of all empirical intuitions.[1.] The knowledge in form resulting from that application is a determination of the limits of knowledge.

Kant's union, after 1770, of conception and intuition alters the doctrines of space as a pure intuition, and as a

1.) The significance of the pure intuition of space in pure Geometry lies in its being at the same time the form of any possible empirical intuition. This makes Geometry a science which determines the limits of knowledge, and makes the pure intuition of space significant in spite of the fact that it is an ens imaginarium.

form of intuition. Space cannot be represented except by drawing it in the mind.[1] The understanding can neither merely intuite space, nor merely conceive it. The understanding can neither merely intuite an object in space, nor merely conceive it. For a human understanding, there is neither space (and time) apart from the conditions of conception, nor categories (rules of conception) apart from intuition. Space is represented only on the occasion of the perception of extended things.

> (B. 348-9.) "Wenn das Licht nicht den Sinnen gegeben worden, so kann man sich keine Finsternis, und wenn nicht ausgedehnte Wesen wahrgenommen worden, keinen Raum vorstellen."[2]

The representation of space depends upon the sensuous manifold. It is with this manifold that the activity of the understanding begins. In pure and applied Geometry, the understanding has this common starting place. Without the sensuous manifold, there could be no pure intuition of space in pure Geometry, and no pure form of intuition of space in applied Geometry. The pure manifold of space

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1.) (A 162-3; B. 203-4) "Ich kann mir keine Linie, so klein sie auch sei, vorstellen, ohne sie in Gedanken zu ziehen, d. i. von einem Punkte alle Teile nach und nach zu erzeugen, and dadurch alleerst diese Anschauung zu verzeichnen. Ebenso ist es mit jeder der kleinsten Zeit (and space) bewandt."

2.) cf. A 196; B 241.

is represented in pure Geometry only in contrast to the sensuous manifold. The pure form of the sensuous manifold is represented in applied Geometry only in contrast to its content, the sensuous manifold. Space without the sensuous manifold is an ens imaginarium.[1] Space is only the form for setting the objects of sense together. If no objects are set together, nothing remains.

> (R. 985.) "Raum und Zeit sind selbst nichts anderes als Formen der Zusammensetzung der Objecte der Empfindung; daher auch, wenn man alle Zusammensetzung da aufhebt, nichts uebrig bleibt."[2]

Unless the synthetic activity of the understanding sets the objects of sense together, there is no space, either as a pure intuition, or as a form of intuition. In both pure and applied Geometry, space is determined by that activity. This doctrine is an outgrowth of the view of Diss. 15. cor. (at the end). There Kant had held that an intellectual activity results in a sensitive whole, in spite of the fact that the activity itself does not begin with the sensitive. After 1770, he believed that the activity of the understanding results in the same sensitive whole, but that it proceeds from the objects of sense (sensuous manifold). Conception and intuition being united, there

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1.) A 291; B. 347.
2.) Also R. 410.

can be no synthetic activity, unless it begin with the sensuous manifold. The nature of space in both pure and applied Geometry is determined by a synthesis[1] which must begin with that manifold. This doctrine destroys the view of the Aes. and of Diss. 15. Space cannot precede the sensuous manifold, if it is determined by a synthesis which begins with that manifold. Space cannot be prior to the occurrence of outer sensation,[2] that is, of outer experience. Space cannot remain if the objects of sense (outer appearances) be abstracted.[3] The whole of space cannot precede and determine the parts,[4] and also be determined by a synthetic activity of the understanding, which begins with the parts. The doctrines of the Aes. remain as doctrines for a possible understanding.[5] For the human understanding, there is the pure intuition of space in pure Geometry only as far as we have the sensuous manifold, and contrast the pure manifold with it. For the human understanding, there is the pure form of intuition, space, in applied Geometry, only as far as we have the sensuous manifold of which space is the form.

1.) A. 163-B. 203; B. 204.
2.) Compare Diss. 15 A. and Aes. I.
3.) Compare Diss. 15 A. and Aes. II.
4.) Compare Diss. 15. cor. (beginning) and Aes. B. III.
5.) Even this is not strictly correct; see below p.126.

In both cases, the parts of space precede the whole. The particular representations of space are under the whole of space and not in it.[1] Space is an aggregate.[2]

I mentioned above the three possible interpretations of Kant's doctrine of a synthetic activity of the understanding from which space is abstracted. We have now come to the root of the difficulty in his conflicting views of space. In his thinking of 1769, and in most parts of Diss. 15., ., and in the Aes., he developed his views of space from the point of view that there is no activity of the mind. Space is presented or given; its relations are contemplated. In two parts of Diss. 15., he said that space is abstracted from an activity of the mind. He made a unity of these doctrines of space in the Diss. by assuming that a wholly intellectual activity can yield a sensitive whole whose conditions are independent of that activity. At that time this reconciliation was possible, for he maintained that there is knowledge based upon a synthesis which has no relation to sense-experience.

1.) Kemp Smith, p. 96. "In principle, the whole precedes the parts; in the process of being brought into existence as an intuition, the parts precede the whole."

2.) pp. 107-108.

After 1770, in accord with his doctrine of knowledge, he maintained, however, that all synthesis must be sensitive. It must proceed from something sensitive,[1.] and yield the sensitive whole of space. This view is apparently merely a slight alteration of the doctrine of an activity of the mind in Diss. 15. cor. (end). There Kant had said that an intellectual activity yields a sensitive whole of space. What could be more justifiable than to say that the same synthetic activity proceeds from the sensitive parts (limits) and yields the concept of a sensitive whole of space? This seemed to Kant to bring space at one stroke under the conditions of conception in accordance with the doctrine that intuitions and concepts are inseparably bound together, and yet to retain the doctrines of Diss. 15. and the Aes. as true of space once it has been yielded by the understanding. This was Kant's view in B. 160n. Space is both the form of intuition and the pure (formal) intuition, but only the latter gives a unity of representation. He said that in the Aes. he had relegated this unity to sensibility in order to remark that it precedes all concepts, although it presupposes a synthesis, (which does not belong to the senses) through which all concepts of space and time are possible. The unity of this a priori intuition belongs

1.) Diss. 5.

to space and time, and not to the concepts of the understanding. For through this unity (in that the understanding conditions sensibility), space and time are for the first time given.[1.] This is a doctrine which -- he believed -- allows a synthesis to yield space, and, at the same time, allows the conditions of unity to remain in space and time as described in Diss. 15. and in the Aes.

This solution conceals, however, a hornet's nest of difficulties. What is the nature of the synthesis which yields space? As I have pointed out, Kant could mean by a synthesis any one of three types. One of these types is a complete return to the view of Diss. 15. cor. (end), and it is compatible with the rest of Diss. 15. and with the four arguments of the Aes. only on the assumption of the dualism between sense and intelligence. The second type (that at B. 160n.) is compatible neither with arguments in the Aes. nor with the theory of knowledge in the Ana.

1.) "Diese Einheit hatte ich in der Aesthetik bloss zur Sinnlichkeit gezaehlt, um nur zu bemerken, dass sie vor allem Begriffe vorhergehe, ob sie zwar eine Synthesis, die nicht den Sinnen angehört, durch welche aber alle Begriffe von Raum und Zeit zuerst moeglich werden, voraussetzt. Denn da durch sie (indem der Verstand die Sinnlichkeit bestimmt) der Raum oder die Zeit als Anschauungen zuerst gegeben werden, so gehoert die Einheit dieser Anschauung a priori zum Raume und der Zeit, und nicht zum Begriffe des Verstandes."

In fact, it is itself quite meaningless. The third is the only type compatible with the theory of knowledge in the Ana., and it contradicts the arguments in the Aes. Let us consider these types in turn.

The first kind of synthesis[1.)] is intellectual and yields the pure intuition of space, which is entirely sensitive. With all of the difficulties of this dualism, certain advantages follow from it. By assuming it, space is determined by two sets of conditions, each of which is complete in itself. Space conforms to the conditions of the intellect so far as it is yielded by an intellectual activity. The completeness of these intellectual conditions is disturbed in no way by the conditions of sense. Space conforms also to the conditions of sense. It is uniquely related to these conditions, as described in Diss. 15., and the four arguments of the Aes. The completeness of these conditions of sense is disturbed in no way by the conditions of intellect. The lynch-pin of the structure of argument in Diss. 15., and in the Aes. is this dualism between sense and intelligence. Granting it, the arguments follow validly. Whatever be the difficulties with the dualism itself, its chief advantage is the support which it

1.) Diss. 15. cor. (end.)

lends to these fundamental arguments. Most of the obscurities in Kant's later teaching result from his attempt to retain the teaching of the Aes. and of Diss. 15., and to discard the dualism upon which it is grounded. Many of his difficulties would have been avoided had he realized that his teaching in Diss. 15., and in the Aes. is based upon the dualism, and must stand or fall with it. Had he faced this fact, and retained the dualism (admitting its incompatibility with his more mature views) the result would have been much less unsatisfactory than his teaching in B. 160n.

I do not mean that the doctrine of an intellectual synthesis, if closely analyzed, turns out to be any more coherent than any other doctrine of synthesis which does not begin with the sensuous manifold. The same criticism which I shall make below of the doctrine of synthesis in B. 160n., can be made with equal cogency against the teaching of Diss. 15. cor (end). Granting two independent sets of conditions, the sensitive and the intellectual, this separation justifies in no way the claim that space may be yielded by an intellectual activity. What is the nature of such an activity? What is the nature of the non-sensitive (i.e., intellectual) elements with which it could

begin? Such a question is in no way answered by the assumption that there is an intellectual, or non-sensitive intuition. Kant grants that such an intuition is possible, but not for man. Such "possibility" is meaningless in the light of the critical doctrine that only such sciences as pure Geometry and Mechanics are concerned with the possibility of knowledge. The justification of the intellectual synthesis in the Diss. rests only upon the assumption that there is analytical knowledge which is concerned in no way with the world of space and time, and that the concepts involved in such knowledge presuppose a synthesis which in no way involves the senses, or even the forms of sense-experience. This assumption possesses the single positive quality in that it is not intended to be compatible with the doctrine of the necessary union between concepts and intuitions. Its weakness lies in the fact that even an intellectual synthesis must begin with something. But this weakness was obscured to Kant (even after 1770) by the more general question: what kind of a content can an intellectual concept have, granting that it has been yielded by a synthesis? Until this question had been more squarely faced by Kant, it is not likely that he would concern himself with the problem of the nature of the elements with which an intellectual synthesis

could begin. Nor is it likely that Kant ever faced this problem; for with his rejection of the dualism of 1770, it disappears. It is almost certain that he never faced the problem of the origin of an intellectual synthesis which in no way concerns either the forms (sensitive elements) or the contents (sensuous elements) of experience until, miraculously enough, it yields the former. This problem is, in fact, supplanted by the similar problem which arises with reference to the doctrine of a second type of synthesis.

After his rejection of the dualism of 1770 Kant formulated the doctrine of a second type of synthesis in the vain hope of retaining the Aes. as a body of teaching, in spite of the doctrine that concepts and intuitions necessarily condition each other. Instead of facing the fact that the rejection of the dualism of 1770 involves the rejection of the arguments of the Aes., Kant formulated in B. 160 n. a doctrine of synthesis which he believed to be compatible with the teachings of the Aes. and the Ana. He believed that in spite of the union between conception and intuition, he could work out a doctrine of synthesis which would leave the arguments of the Aes. intact. The result is a meaningless doctrine compatible neither with

the union of conception and intuition nor with the arguments of the Aes. In B. 160n., he said that a synthesis yields a sensitive whole of space. This synthesis cannot be intellectual, for there can be no isolation of conception from intuition. It must, therefore, not only yield the sensitive whole of space, but it must begin with something sensitive. Yet it does not -- Kant claimed -- begin with the sensuous manifold. The obvious question (which he apparently never faced) is: If such a synthesis is not intellectual, and does not begin with the sensuous manifold, with what does it begin? 1.) There may be some justification for Kant's omission of this question in 1770, when his isolation of sense and intellect left open -- as he admitted -- many questions concerning the nature of the latter. But in 1781 it was quite different. His union of conception and intuition involves necessarily the problem of the nature of the synthetic activity of the mind which yields space. His insistence that space is yielded by a synthesis is equaled only by his persistent omission of the question concerning the nature of the elements with which such an activity could begin. Had he faced this question, he would have realized that a synthesis must begin either

1.) Compare Kemp Smith, p. 97. "Nor does (Kant) show what the simple elements are from which the synthesis of apprehension and reproduction in pure intuition might start."

with the sensuous manifold, in which case, it is preceded and determined by that manifold, or it must begin with the very thing it is supposed to yield, namely, space. In the latter case, the doctrine of synthesis is not only incompatible with the teachings of the Aes., but it is quite meaningless. If a pure manifold, space, is given with which a synthesis may begin, then space is not derived from the synthesis, but precedes and determines it. Either there is no pure manifold given with which a synthesis can begin, or space is prior to all synthesis. In the former case, there can be no synthesis unless it is intellectual, (which means a return to the dualism of 1770) or unless it begins with the sensuous manifold. A pure manifold is "space" as much before a synthetic activity as afterwards.[1] A particular space is "space" as much as a greater (or smaller) particular space. Granted a pure manifold, then space does not presuppose a synthesis of any sort. Granted a pure manifold, then space is prior to sense-experience, the parts of space are limits, and all particular representations of space are <u>in</u> it and not <u>under</u> it. To assume a manifold (other than the sensuous manifold) from which a synthetic activity can begin is to restate the arguments of Diss. 15. and of the Aes. which preclude any synthetic activity. To assume that a synthetic

1.) See above pp. 91-92.

activity yields space is to deny that a manifold from which it began is pure. A synthetic activity must begin with the sensuous manifold.

The only possible type of synthesis of the understanding which can yield space is one which begins with the sensuous manifold. Such a synthesis is the only one which is compatible with Kant's teaching in the Ana. It is an essential part of that teaching. This doctrine of synthesis contradicts the four arguments of the Aes.[1.] If it begins with the sensuous manifold and yields space, then the sensuous manifold is prior to space. Space is not prior to but later than sense-experience. This contradicts Diss. 15. A., and Aes. I. If the objects of the sensuous manifold are abstracted, nothing remains.[2.] This contra-

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1.) Hartmann, p. 154, quoted by Vaihinger II, p. 228. "Diese Erklaerungen (in B 160n.) genuegen, um Kants Schlussfolgerung in ihr Gegentheil zu verkehren. Wenn der einige Raum als gegebenes Ganzes erst Product einer vom Verstande ausgefuehrten Synthese des raeumlichen Mannigfaltigen ist, so ist er spaeter als diese, aber nicht frueher; es muessen dann die durch die sinnliche Anschauungsform allein aus der Empfindung formirten endlichen Anschauungen (das raeumliche Mannigfaltige) das fruehere sein, aus welchem erst der einige Raum sich bilden kann, und nimmermehr koennen sie ihrer Enstehung nach blosse Einschraenkungen dessen sein, was erst vermittelst ihrer zu Stande kommen kann, indem der Verstand sich dieses ihm gegebenen Stoffes combinatorisch bemaechtigt. Kant hat leider nicht bemerkt, dass er in dieser Anmerkung zur 2. Aufl. der Analytik selbst seine fruehere verkehrte Auffassung ueberwunden und berichtigt hat."

2.) See above p. 119.

dicts Diss. 15. A. and Aes. II. If space is yielded by a synthetic activity of the understanding which begins with the sensuous manifold, then the parts of space are prior to the whole, and determine it. The whole is merely an aggregate. Particular representations are not <u>in</u> space, but <u>under</u> it, just as usual particulars determine usual general concepts. This contradicts Aes. B. III., and B. IV. This is the view of space to which Kant came if his doctrine of a synthesis is to accord with his theory of knowledge in the Ana. Unless there is a dualism between conception and intuition, as in Kant's view in the Diss., the sensuous manifold is prior to space, space does not remain after the objects of sense have been abstracted, and the parts of space precede and determine the whole. The only other possibility for Kant was a return to the view of the Diss., and a separation of conception and intuition into independent faculties. Such a doctrine of synthesis as in B. 160 n. is meaningless, and is compatible neither with the teaching of the Aes. nor with that of the Ana.

Cassirer points out a unity in the diversity of Kant's doctrines in the Aes. and Ana. Intuitions and con-

ceptions -- Cassirer says -- are inseparable, but they may be treated as separated. In the synthetic construction of knowledge, an intuition conditioning an object can in no way be separated from the functions of thought. On the other hand, there is no logical contradiction involved, no violation of the highest principle of analytical judgments in treating them as separated.[1.] The diversity in the Critique lies in the fact that in the Aes., intuitions and conceptions are treated as separated, while in the Ana., they are treated as joined. Cassirer forgets that there is no more evidence in the Aes. that Kant believed that they are actually joined and treated them as if they could be separated, than there is evidence in the Ana. that Kant believed that they are separable and treated them as if they were necessarily joined.

There is some justification for interpreting the doctrines of the Aes. from the point of view of the Ana. The latter

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1.) (II. p. 700.) "Die Anschauung kann im synthetischen Aufbau der Erkenntnis, sofern also durch sie ein Gegenstand gegeben und bestimmt werden soll, die Denkfunktion freilich in keiner Weise entbehren; dagegen bedeutet es zum mindesten keinen logischen Widerspruch, bedeutet es keinen Verstoss gegen den obersten Grundsatz aller analytischen Urtheile, sie von ihr losgeloest zu denken."

is a more mature one. Kant's development was not, however, a steady advance from the teachings of the Aes. to those of the Ana. His original contrast in 1769 between the concept of space and the usual general concepts is more compatible with the view of the Ana., than with the view of the Aes. As Cassirer remarks,[1.] Kant believed space to be a concept of the understanding and a principle of synthesis before he believed it to be a pure intuition. The doctrines of the Ana., are the culmination (years later) of Kant's search in 1769 into the differences between general concepts, of a certain type, and space and time whose nature involves certain synthetic processes of the mind. In Diss. 15. he relegated to the Corollary the doctrine of a synthetic activity which determines space because/his discovery concerning the nature of mathematical truth led him to overstate his doctrine of pure intuition. In the Aes., he worked out the doctrine of pure intuition, ignoring completely his doctrine of a synthetic activity which determines space. In the Ana., with the surrender of his dualism, and his return to the theory of an activity of the mind which yields space, he returned to a view quite similar to the/ one that he had held in his investigation in 1769 of space as a concept of

1.) Cassirer II, p. 625n.

the understanding. His teaching in the Ana. gives us the first satisfactory logical treatment of space in contrast to general concepts. In this way, his views in the Ana. may be considered the culmination of his thinking of 1769. The doctrines in the Ana. may be considered Kant's more mature ones, but this does not mean that he arrived at all of them after writing the Aes.

Cassirer realizes that in the Aes. Kant says one thing, and in the Ana., he says another. The genuine separation of concepts and intuitions in the Aes. becomes the logical correlation in the Ana.

> "So loest sich die anfaengliche Trennung von Anschauung und Begriff immer deutlicher in eine reine logische Korrelation auf. Die Unterscheidung, die die transzendentale Aesthetik an die Spitze stellt, betrifft zunaechst nur die Absonderung von den gewoehlichen Gattungsbegriffen." (II. 698.)

If we could overlook the wording of the Aes. and Diss. 15., and interpret Kant's teaching solely in the light of his conclusions in the Ana., we might readily claim that he was concerned only with the logical nature of space from the time of his original contrast of space and general concepts in 1769 until the publication of the Critique. Such an interpretation would ignore the discovery in 1769-70

that truth in Geometry and Mechanics is a priori and synthetic, as well as the wording of the arguments in Diss. 15., and the Aes. We cannot claim that there is a unity of the doctrines of space in the Critique whatever significance the teachings of the Aes. retain if viewed as a logical treatment of the concept of space. In denying this unity to the Critique, I wish to undervalue in no way the interpretation which Cassirer gives to the Ana. I wish merely to emphasize that, after all, it is only the point of view of the Ana. of which he speaks. His treatment of Kant's development takes account of many of Kant's changes of doctrine. From the point of view of the Ana., the conditions of conception are dependent upon the intuitions of space and time, and the conditions of sensibility are dependent upon the categories. In the act of knowledge, the categories are bound to intuition. We may abstract from the conditions of knowledge, and arrive at the conditions of sensibility apart from the conditions of knowledge. This doctrine is important whether or not the conditions of sensibility referred to here are the conditions described in the Aes. To make such an abstraction, laying bare the conditions of sensibility, is a valid procedure logically.

In this way, we do not set up the conditions of knowledge, but only certain conditions which knowledge cannot contradict. The contrast between "man" and "space" is between two determinants of our knowledge. In the contrast, one is as far removed from the conditions of knowledge as the other, yet knowledge cannot go counter to the results of this contrast. Such a contrast may be called knowledge in form. It bears the same relation to knowledge as pure Geometry does to applied, or as pure Mechanics to applied. It will be remembered [1.] that pure Geometry, according to Kant's teaching after 1770, is knowledge in form, resulting from the application of concepts to the pure intuitions of space and time. It is not knowledge, but by means of it the limiting conditions of knowledge are determined. We may describe "space" as isolated from the conditions of conception. We may describe the categories as isolated from the conditions of sensibility. Such descriptions do not concern the conditions of knowledge, but only certain conditions which knowledge cannot contradict. Such descriptions are the "pure Geometry" of knowledge. This is the significance of the arguments of the Aes. if they be interpreted only from the point of view of the Ana. This

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1.) See above, p. 113 .

is the unity which can be pointed out in the diversity of Kant's doctrines in the Aes. and Ana. The value of such an interpretation is obvious. Its disadvantage is that by simplification it omits as much of Kant's teaching as it includes. As Professor Kemp Smith says: "The Critique contains too great a variety of tendencies, too rich a complexity of issues, to allow such a simplification." Treated from neither particular point of view, the doctrines of the Ana. remain in fundamental contrast with those of the Aes. 1.)

Kant failed to unify his doctrines of space in the Critique. Space is either prior to the manifold of sense:- in which case, it is not determined by any kind of synthesis; or space is determined by synthesis which begins with the prior existing manifold of sense. The first is the teaching of Diss. 15. A., and Aes. I.; the second is the teaching of the Ana. Either space remains after the objects of sense have been abstracted, or nothing remains after they have been abstracted. The first is the view of Diss. 15. A., and Aes. II; the second is the view of the Ana. Either the whole of space determines the parts:- in which case, there is no

1.) Kemp Smith, p. 102.

synthesis; or the parts precede the whole, and it is determined by a synthesis of them. If the whole (aggregate) has been reached by a synthesis, then it cannot precede the parts, but is preceded and determined by them. If the synthesis is carried further, a new whole results, and what had been previously considered as the whole, is found to have been merely an aggregate, determined by the extent of the previous synthesis of the parts. Space cannot presuppose an activity of the understanding (even in pure Geometry) unless the parts determine the whole. This contradicts Diss. 15. C. n. and 15. cor. (first paragraph).[1.] Space cannot presuppose an activity of the understanding unless particular representations of space are under "space" and not in it.[1.] This contradicts Aes. B. IV.

Kant was trying to hunt with the hounds and run with the hare. He was trying to show that space is prior to the sensuous manifold, and yet that space presupposes a synthesis. He was trying to show that the parts of space are dependent upon the whole, and yet to say that the whole of space is determined by a synthesis of the parts. It cannot be done. His attempt to combine these views failed. His only alternative was the view of the Diss. that the

1.) Vaihinger, II. 170; 228-229.

conditions of synthesis can be entirely intellectual, and that the conditions of the resulting whole can be entirely sensitive. On such a view, sense and intellect are dogmatically assumed to be separate and complete in themselves. But in the Ana. where they are bound inseparably together, the assumption is false.

These conflicting views of space in the Critique make clearer the two views of space in Diss. 15. In 1770, Kant could answer Euler's question in two ways. Space is not abstracted from sensation; in reference to the sensible world, it is not abstracted at all. There is, however, an intellectual synthesis which yields it. It is determined by this synthesis. Kant reconciled these two answers by means of the dualism of 1770. To know the sensible world, the mind requires only intuitions. Knowledge of the sensible world is synthetic, and not analytic. On the other hand, to know the intelligible, real world, the mind requires only concepts. Knowledge of the intelligible world is analytic and not synthetic. These two kinds of knowledge do not overlap.

In every section of Diss. 15. save two [1.] Kant treated the pure intuition of space as presented or contemplated, and not constructed. In two parts [1.] he treated it as

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1.) See above pp. 104-107.

yielded by an activity of the mind. By following out this assumption, he discovered the weakness in his dualism. At the end of the Corollary, he suggested that the concepts of space and time, which are not abstracted from sense, and which are not gained by reflection (like the usual general concepts) are abstracted from an activity of the mind on the occasion of sensation. He assumed that an intellectual activity can yield a sensitive whole. After 1770, he realized that this is impossible. He realized, however, that the fault lay not in the assumption of an activity of the mind, but in the deeper assumption of a thoroughgoing dualism. The assumption of an activity of the mind must -- he realized -- be retained, but the dualism rejected. He could not isolate conception and intuition. There is an activity of the understanding which begins with the manifold of sense and which determines space. The understanding represents space as a pure manifold (in pure Mathematics) in contrast to the sensuous manifold; and as a pure form of the sensuous manifold (in applied Mathematics.). At the end of the Corollary to Diss. 15., Kant was correct in saying that there is an activity of the mind. He was wrong only in regard to its nature. The rest of Diss. 15. is based upon the false

assumption that there is no activity of the mind. Diss. 15., as a whole, is based upon the false assumption that conception and intuition can be isolated. The doctrine that space is prior to sensation is based also upon this false assumption. The doctrine that the whole of space determines the parts is based also upon it. The same is true of the doctrine that an infinite number of representations are in space instead of under it. Once Kant corrected this assumption, and joined conceptions and intuitions, the representations of space and time are subjected to the conditions of conception as well as to the conditions of sense. He could preserve the doctrines of space in the Aes. only by reverting to the point of view of the Dissertation and separating conception from intuition. He would never have done this. He retained the views of the Aes., either because he did not realize that he had contradicted them; or because, realizing it, he knew that the alteration required was too far-reaching to be undertaken.